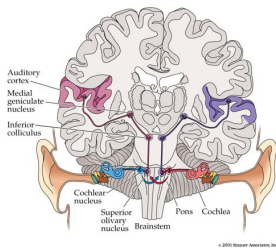


We do interferometry all the time. . .

Ascending auditory pathways



“Significant number of nerve fibers cross the brain and make connections with neurons on the side opposite from the side of the ear in which they begin. This happens very early on in the auditory system. Inter-aural comparisons are an important source of information for the auditory system about where a sound came from.”

{David Heeger's lecture from

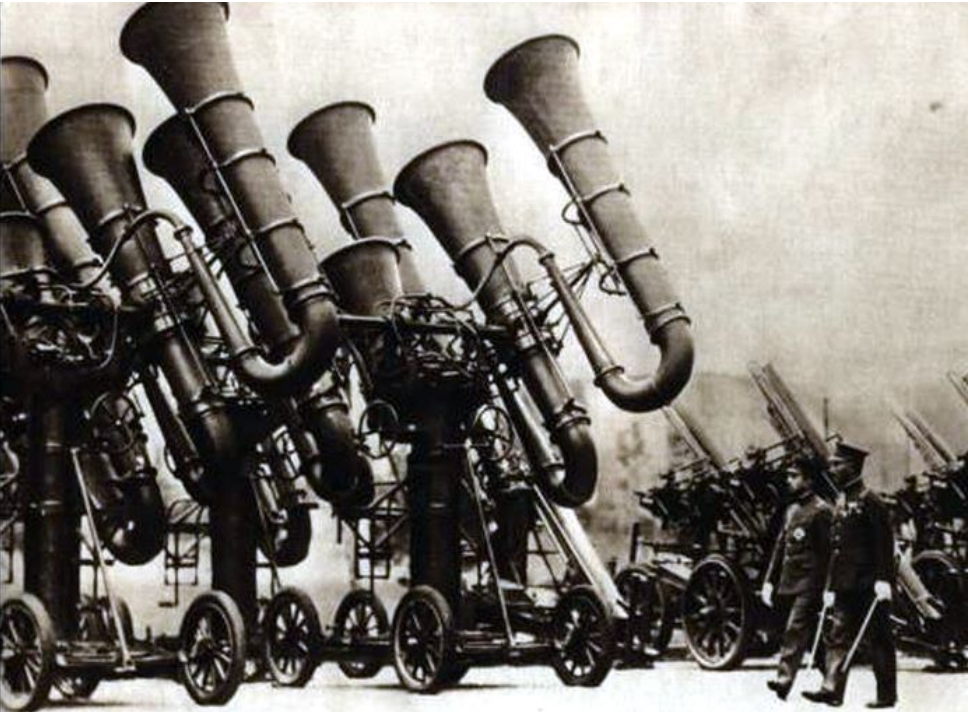
<http://www.cns.nyu.edu/~david/courses/perception/lecturenotes/localization/localization.html>}

And animals too

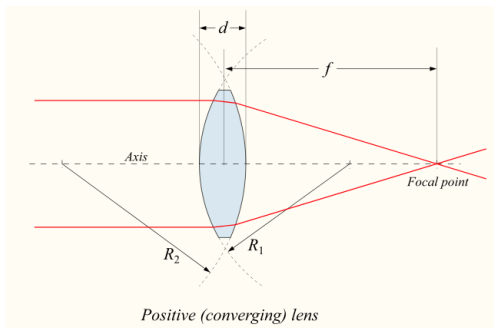


Long eared bat, korvayökkö, *Plecotus auritus*





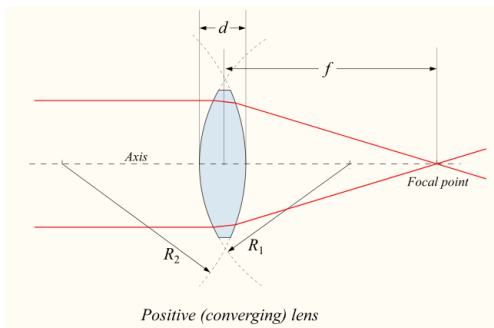
An Introduction to Interferometry



Interferometry is essentially the same thing that happens in an eye or a camera: the electromagnetic field is Fourier transformed by the lens to produce an image. Nothing more, nothing less.

See e.g. https://en.wikipedia.org/wiki/Fourier_optics#Fourier_transforming_property_of_lenses

A Light Introduction



In interferometry, we discard the lens (or mirror), take samples of the aperture field and produce the image mathematically. This works in radio because wavelength is long enough. It's that simple.

Resolution of an optical device

Any optical device is fundamentally limited by **diffraction**

Due to diffraction, the image of a point source is not a point image, but a blurry point-like shape called **Point Spread Function (PSF)**



The PSF of an optical device

The PSF of an instrument is equal to the Fourier transform of the aperture's autocorrelation (in units of λ)

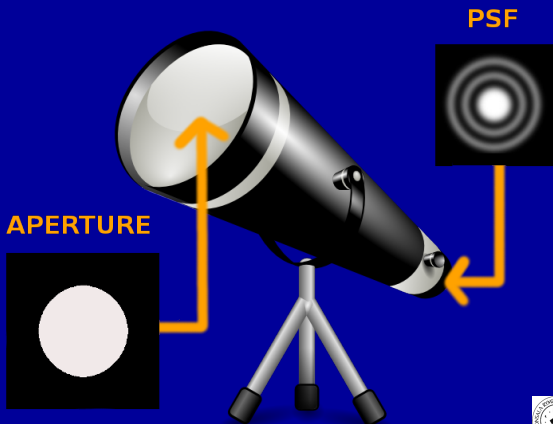
$$\text{PSF} = \mathcal{F}(\text{APERTURE} * \text{APERTURE})$$



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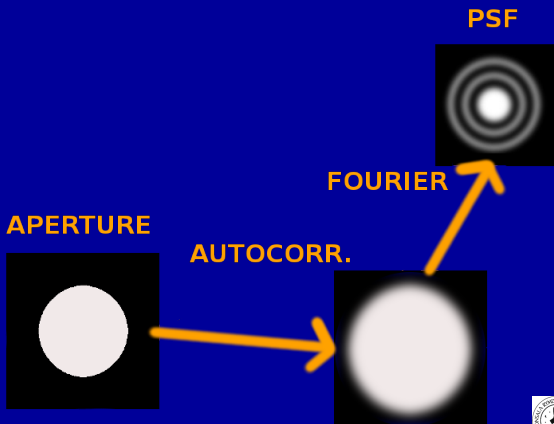
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The size of the PSF depends on the wavelength and the aperture size:

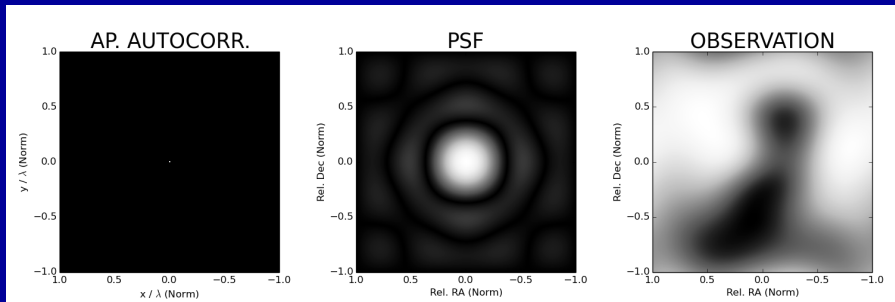
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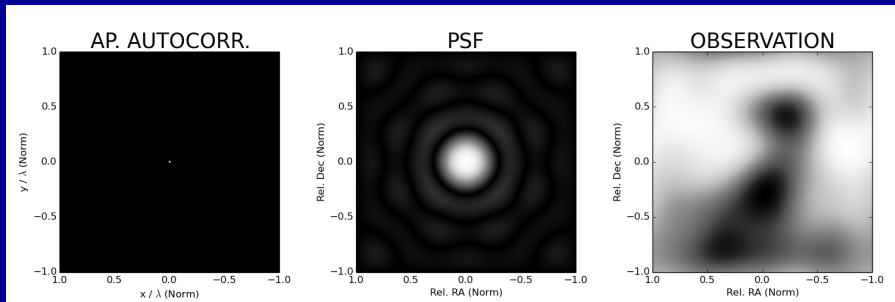


The resolution increases if the aperture increases (in units of λ)

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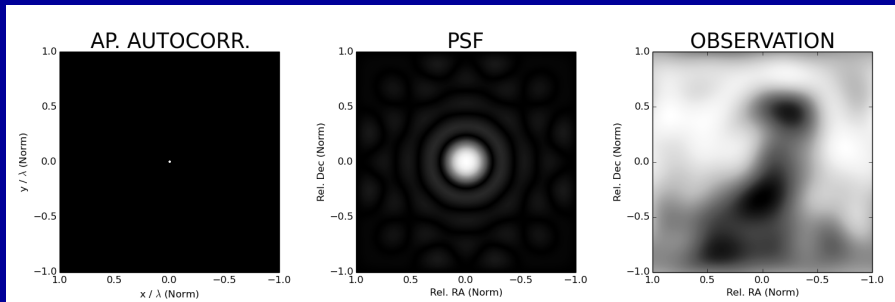
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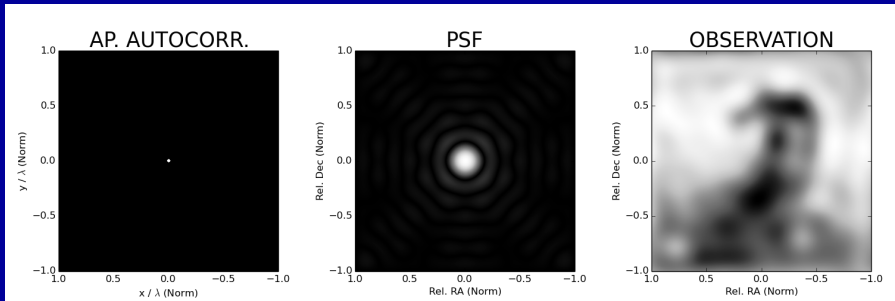
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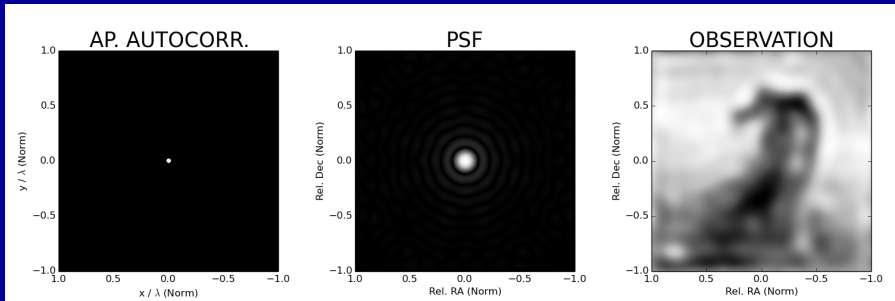
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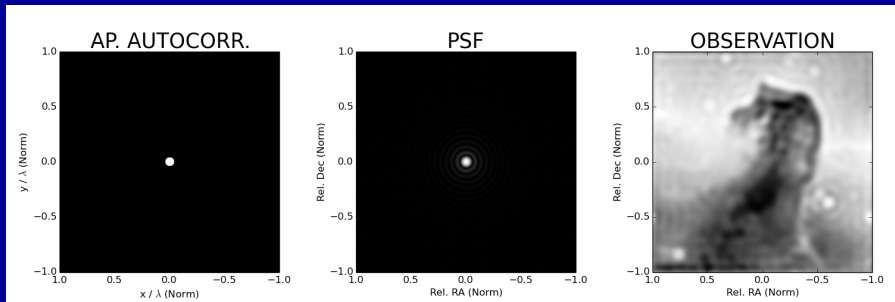
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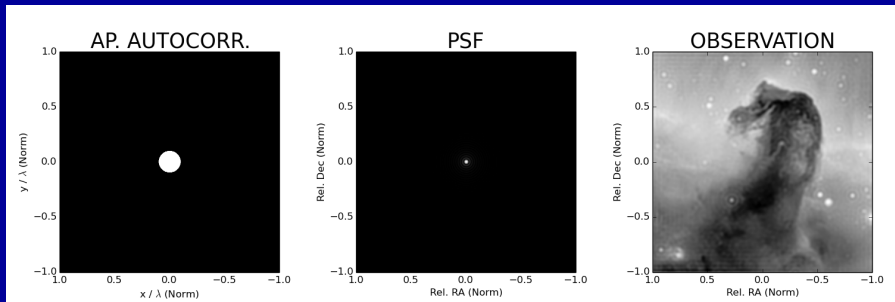
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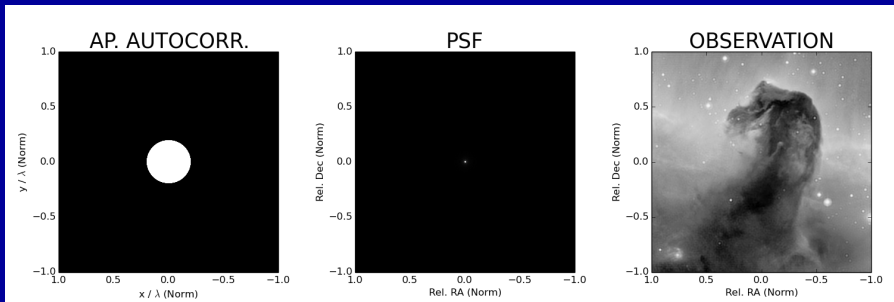


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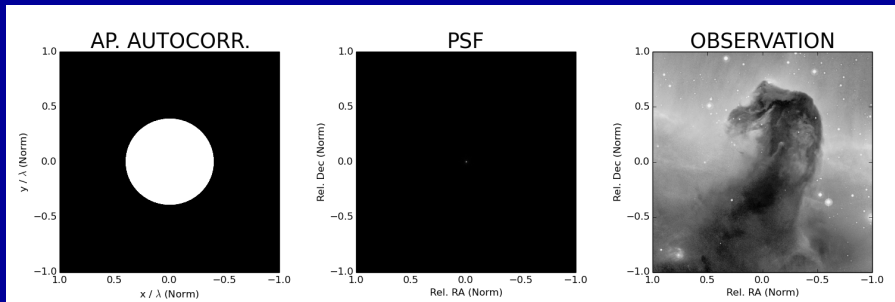


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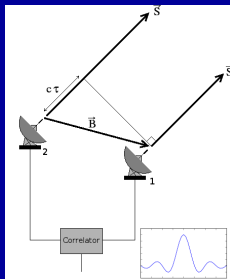
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The two-element interferometer

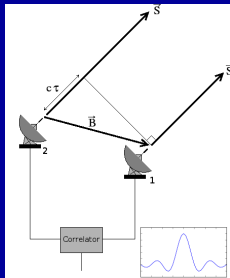


Observing with a two-element interferometer



A BASIC OBSERVABLE OF AN INTERFEROMETER IS THE
CROSS-SPECTRUM (a.k.a. **COMPLEX VISIBILITY**):

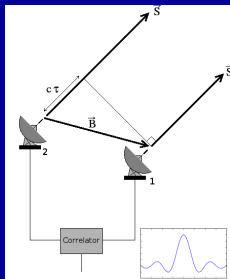
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$$E_i(t)$$

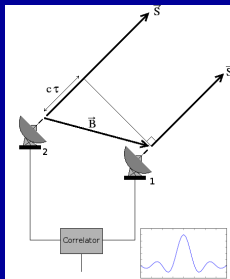
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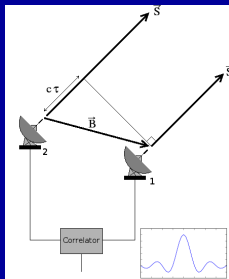
Observing with a two-element interferometer



A BASIC OBSERVABLE OF AN INTERFEROMETER IS THE
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$$E_i(t) \rightarrow S_i(\nu) = \mathcal{F}(E_i(t))$$

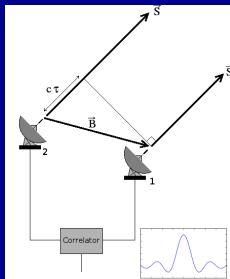
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Observing with a two-element interferometer



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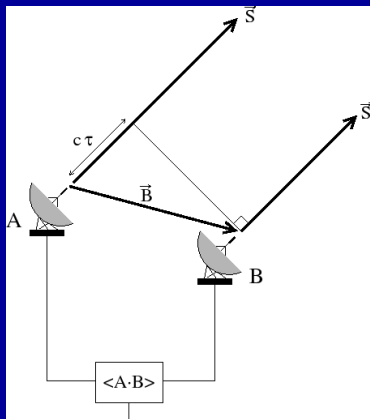
$$E_i(t) \rightarrow S_i(\nu) = \mathcal{F}(E_i(t)) \rightarrow V_{ik}(\nu) = S_i(\nu) \times S_k^*(\nu)$$

Notice the similarity with the **power spectrum** used in **single dish**:

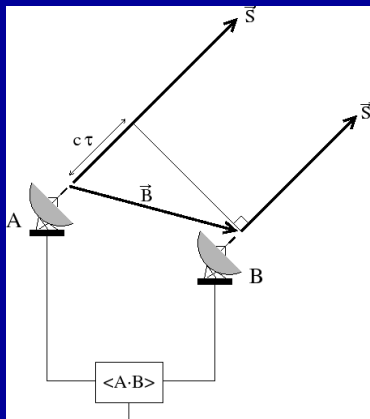
$$P_i(\nu) = S_i(\nu) \times S_i^*(\nu) = |S_i(\nu)|^2$$



Cross-spectrum of a monochromatic point source



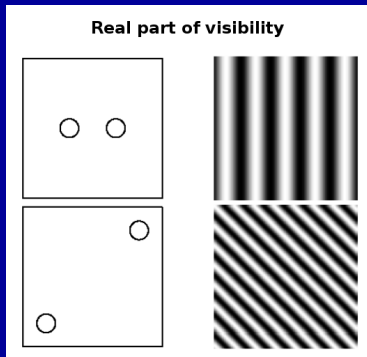
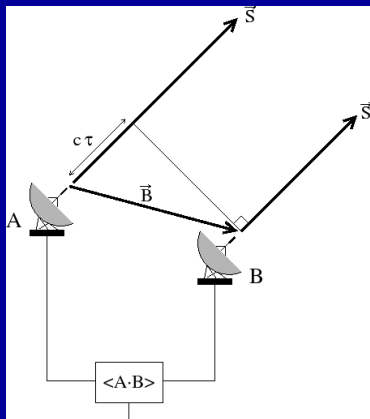
Cross-spectrum of a monochromatic point source



$$\tau = \frac{\vec{B} \cdot \vec{S}}{c}$$



Cross-spectrum of a monochromatic point source



$$\tau = \frac{\vec{B} \cdot \vec{S}}{c} \rightarrow V(\nu_0) = I \times (\cos 2\pi\nu_0\tau - j \sin 2\pi\nu_0\tau)$$



Visibilities of extended sources

For extended sources, the overall measured visibility is the integral of the source structure times the response.

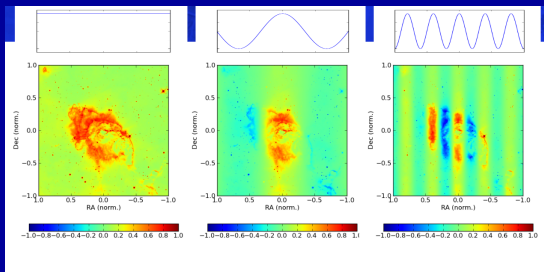
$$V(\vec{B}, \nu_0) = \int I_{\nu_0}(\vec{S}) \exp\left(-2\pi j \frac{\vec{B}\vec{S}}{\lambda_0}\right) d\vec{S}$$



Visibilities of extended sources

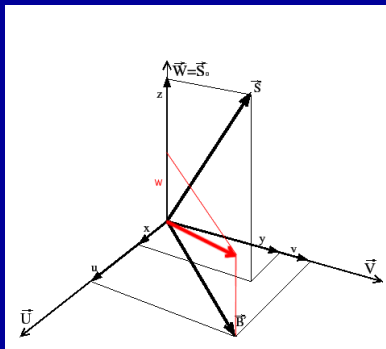
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For small fields of view, this is just the integral of the source structure multiplied by a *complex fringe*.

The Full Interferometer Equation



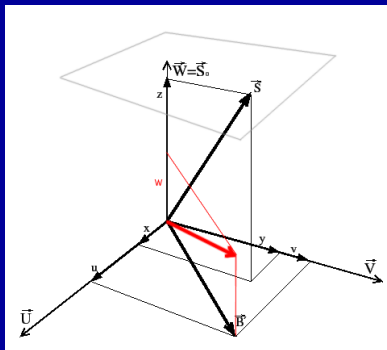
$$V(u, v, w) = \int_{x,y} I_{\nu}(x, y) e^{-\frac{2\pi j}{\lambda}(ux + vy + wz)} \frac{dx dy}{z}$$

where $z = \sqrt{1 - x^2 - y^2}$. The term wz can be made zero by adding the right time delay between the antennae. This equation reduces to a **Fourier Transform** relationship between $I(x, y)$ and $V(u, v)$, for small x and y .

Notice the λ dividing in the exponent. If u and v are given in units of λ , the exponent becomes $-2\pi j(ux + vy)$, i.e. the exact definition of Fourier transform.



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The two-element interferometer. Summary

- **Diffraction limit** is one of the main limitations of optical devices. It is specially important at long wavelengths, where **extremely large apertures** are needed for a reasonable angular resolution.
- The **cross-spectrum** between the signals of two telescopes (observing the same source) is directly related to the Fourier transform of the source structure, computed at the point of **Fourier space** given by the **baseline projection** of the telescopes into the **UV plane** (i.e., the plane perpendicular to the source direction).
- This result will allow us to **synthesize large apertures** through **interferometric observations** (see next lecture).



Aperture Synthesis

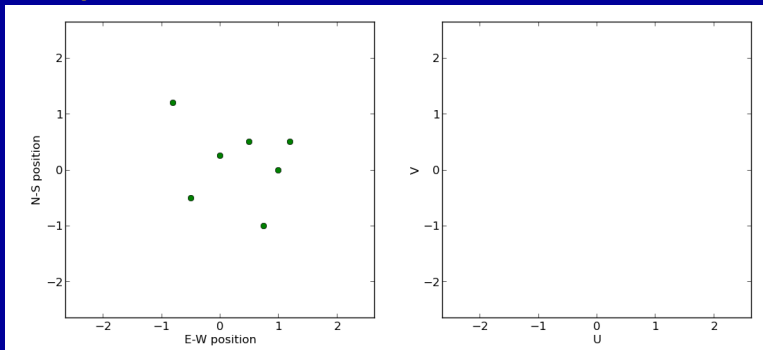


Multi-element interferometers I:

Snapshot observations

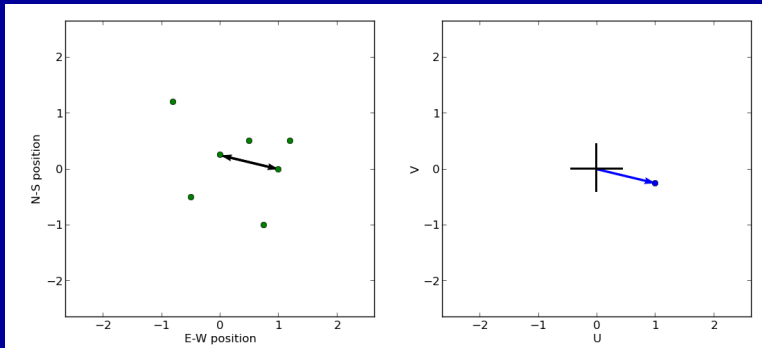


The UV plane



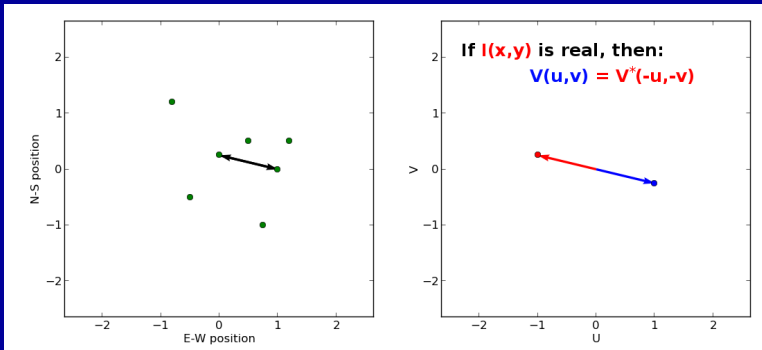
Each pair of telescopes is separated by its **baseline** vector. The measured visibility is the Fourier transform of the source structure measured at the points of the UV plane given by the baseline. **A multi-element interferometer is, hence, just a set of many two-element interferometers!**

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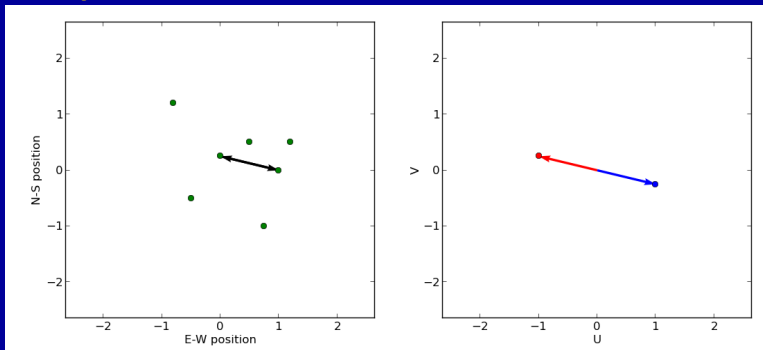
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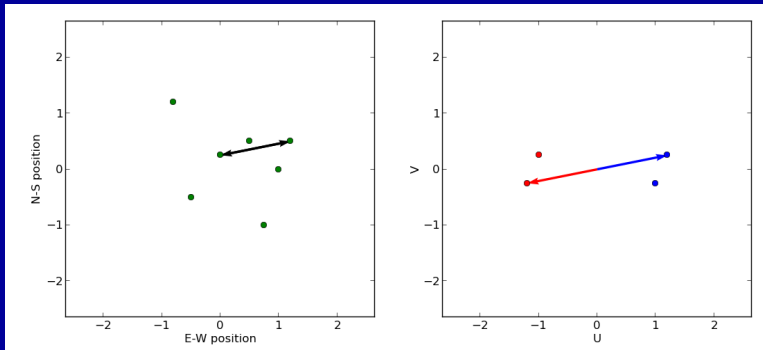


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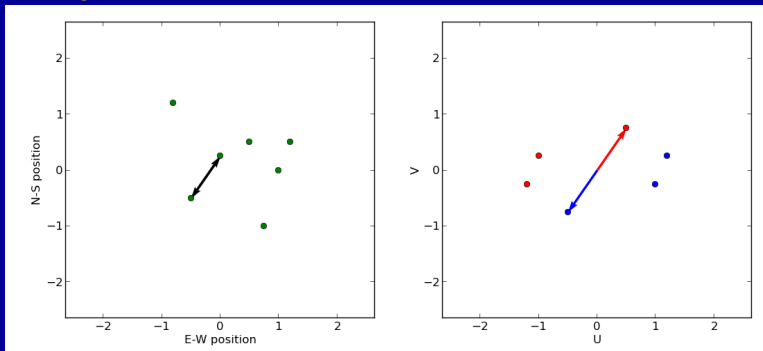
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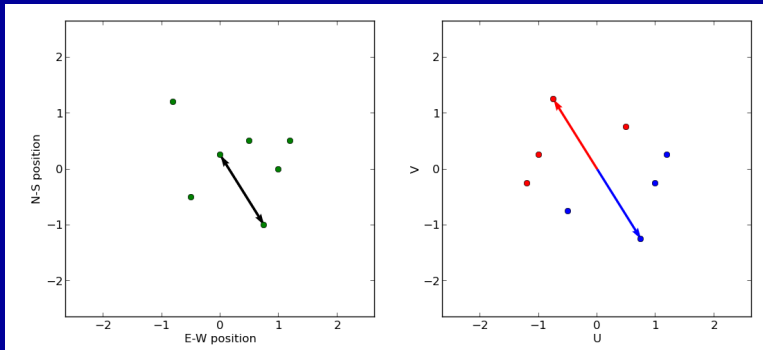


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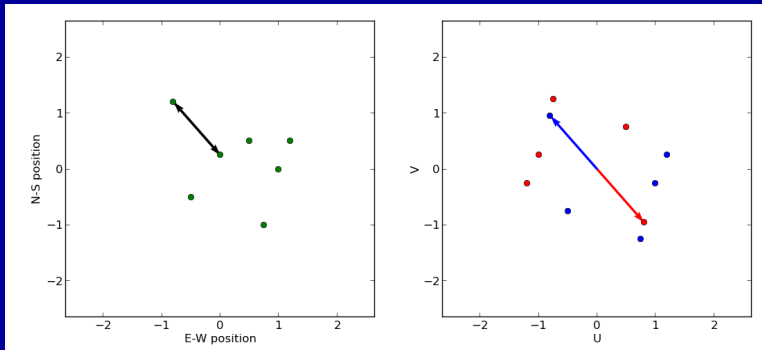
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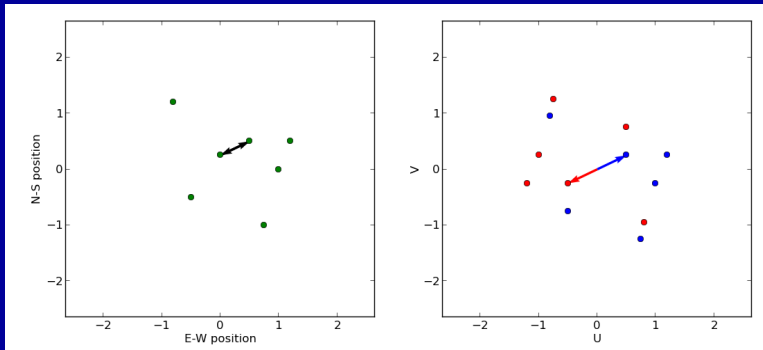
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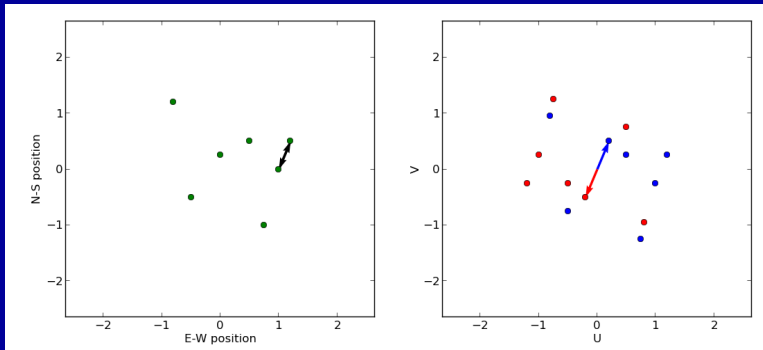
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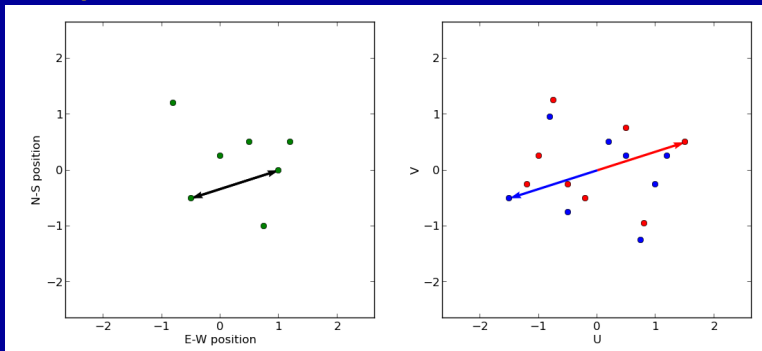
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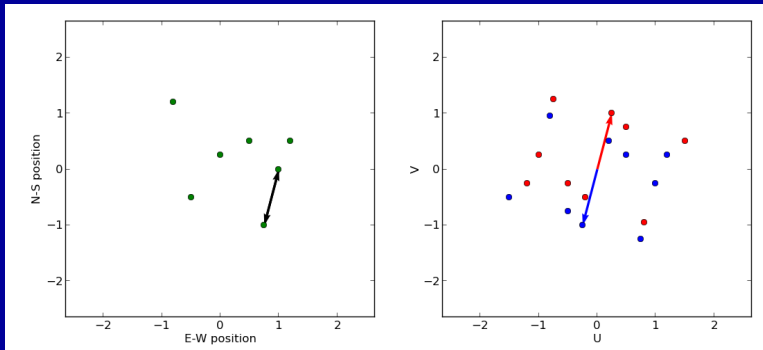
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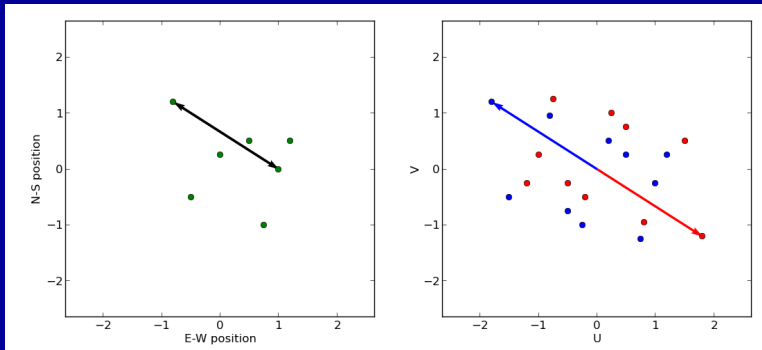
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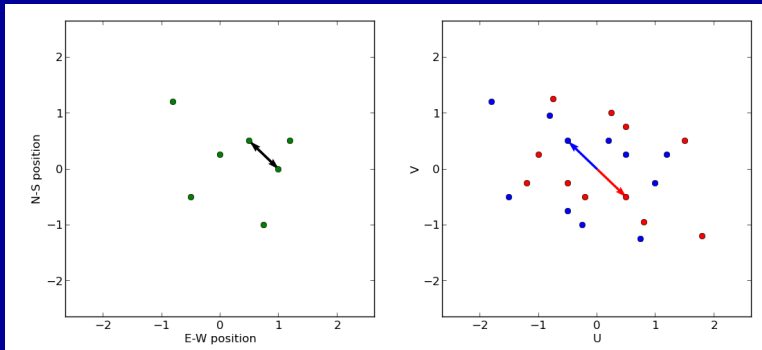
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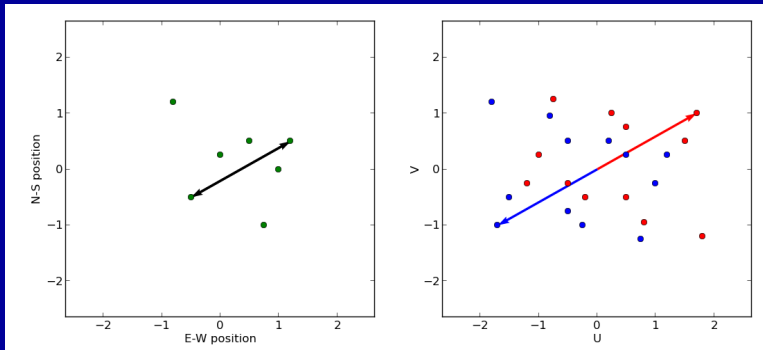


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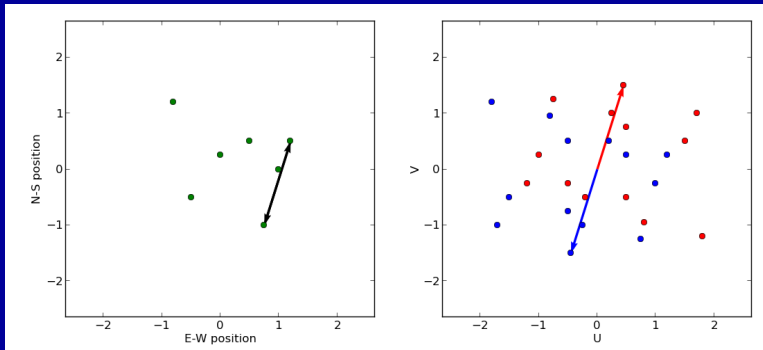
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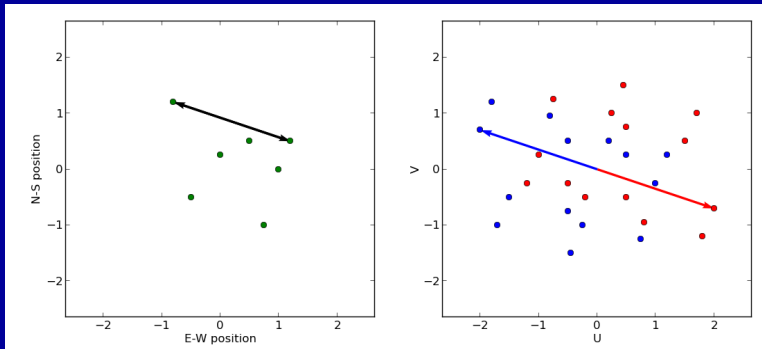
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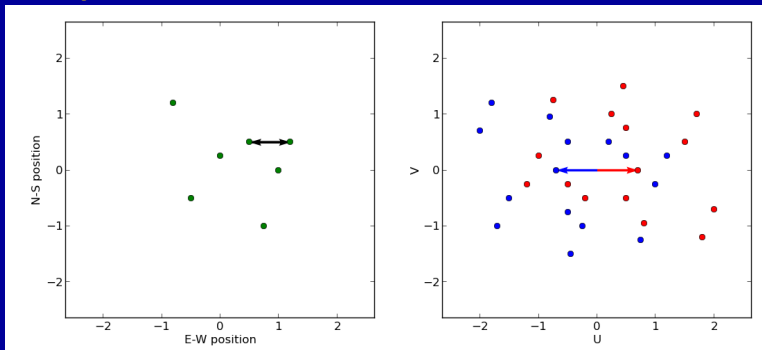


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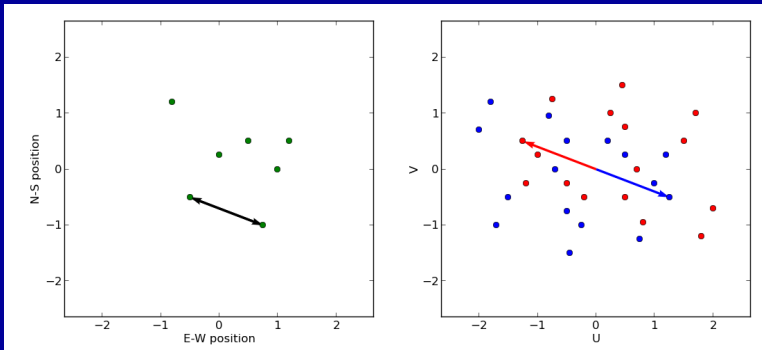
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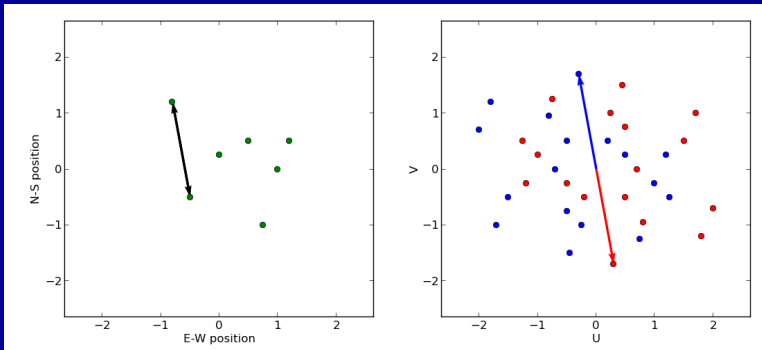


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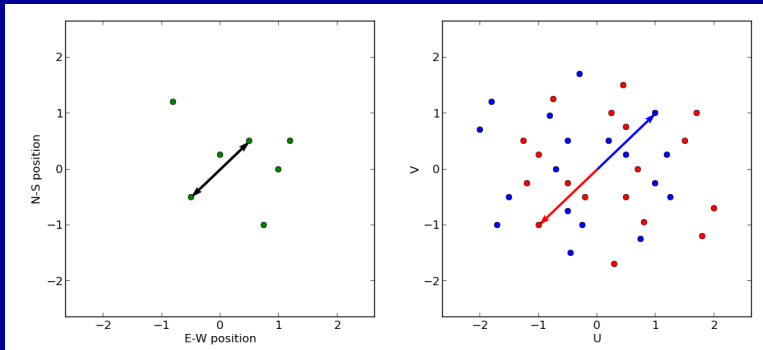
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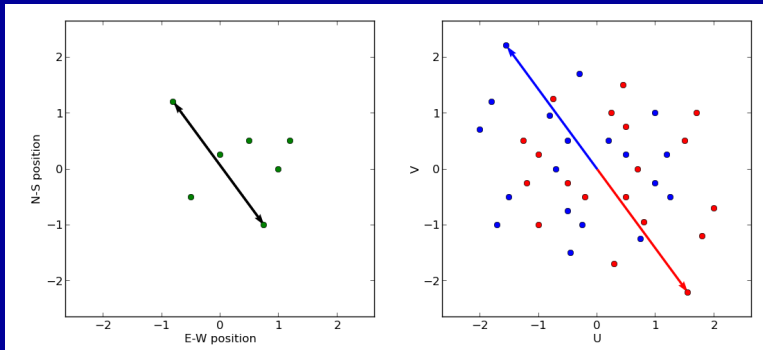
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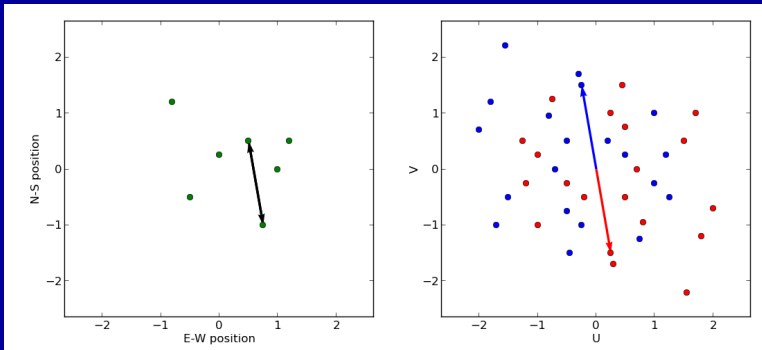
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The UV plane



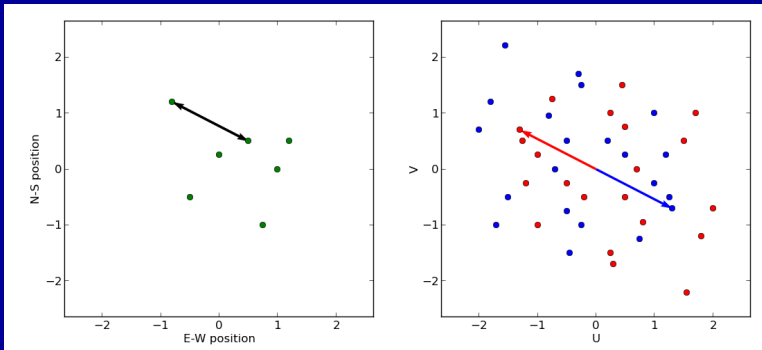
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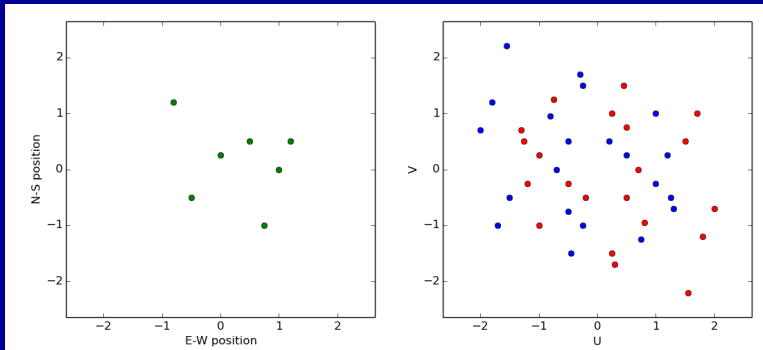
The UV plane



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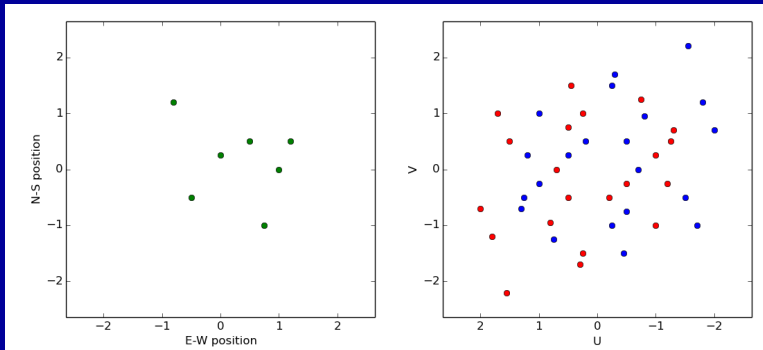
The UV plane



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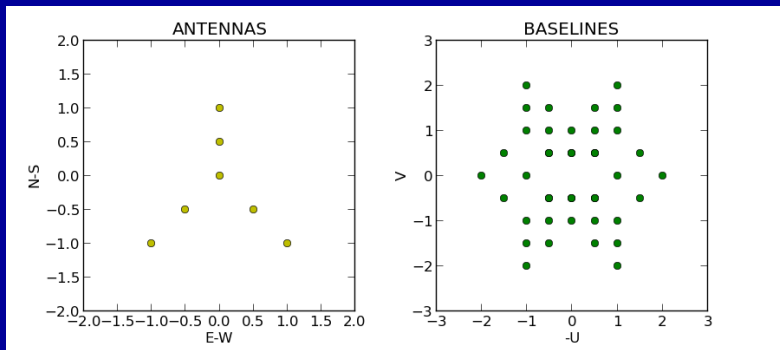


Each pair of telescopes is separated by its **baseline** vector. The measured visibility is the Fourier transform of the source structure measured at the points of the UV plane given by the baseline. **A multi-element interferometer is, hence, just a set of many two-element interferometers!**

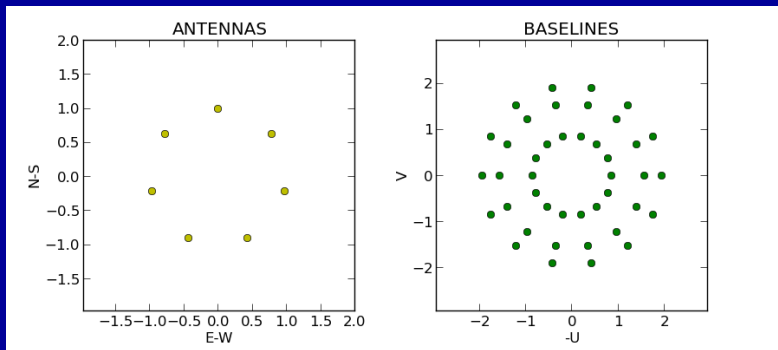
BEWARE! The **U** axis must point to **East** (like the **Right Ascension**!)



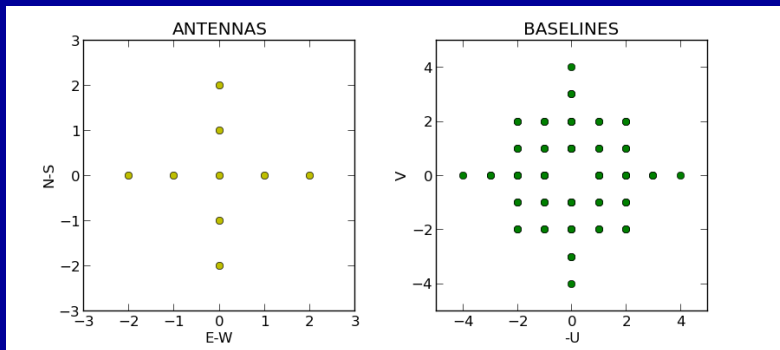
The UV coverage



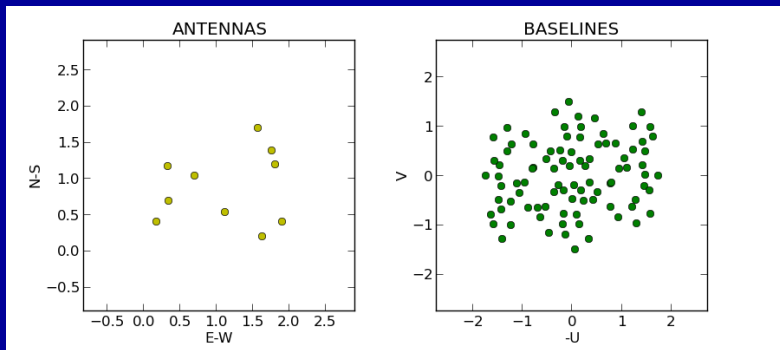
The UV coverage



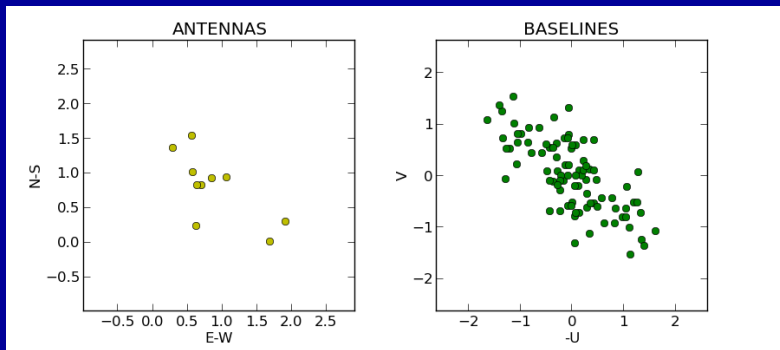
The UV coverage



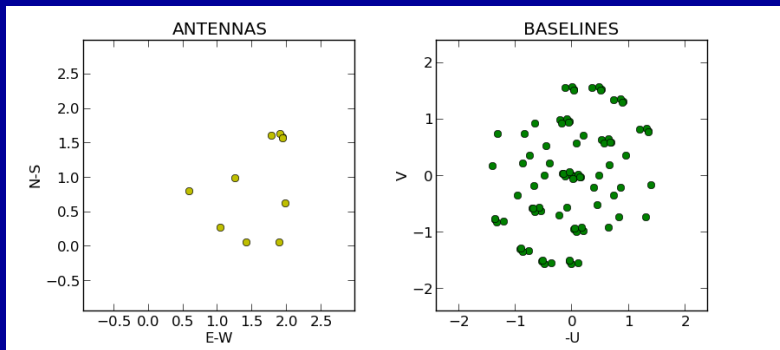
The UV coverage



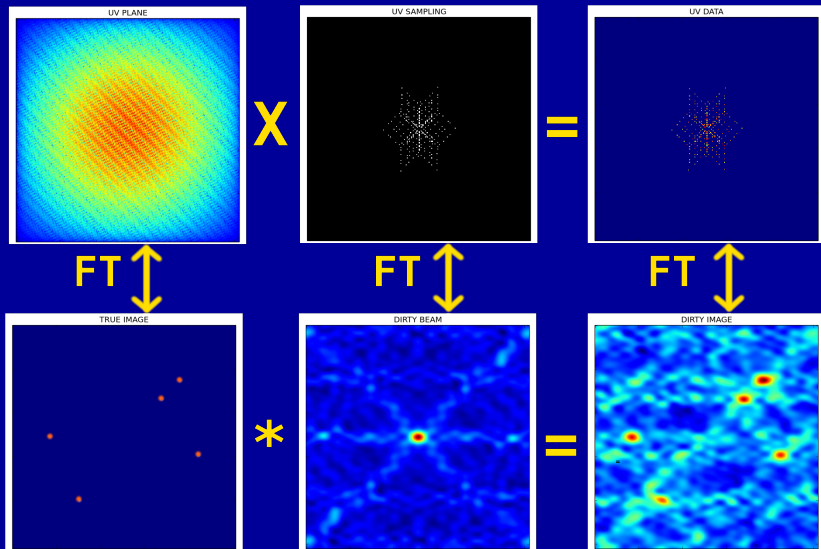
The UV coverage



The UV coverage

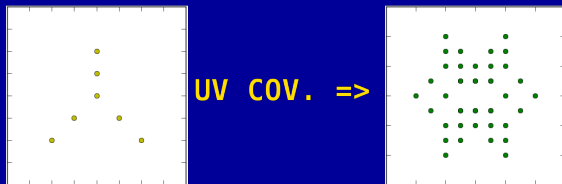
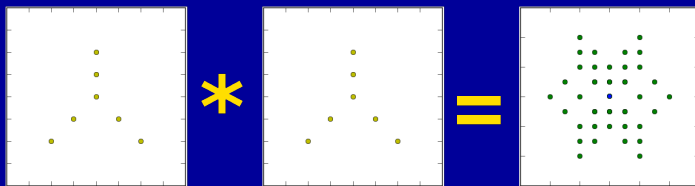


Synthesizing aperture. UV-coverage and PSF.



Synthesizing aperture. UV-coverage and array.

The UV coverage in a snapshot is equal to the **autocorrelation** of the array configuration as seen by the source (but removing the zero-spacing term)



Synthesizing aperture.

The interferometer seen as a single telescope

$$\text{PSF} = \mathcal{F}(\text{APERTURE} * \text{APERTURE})$$



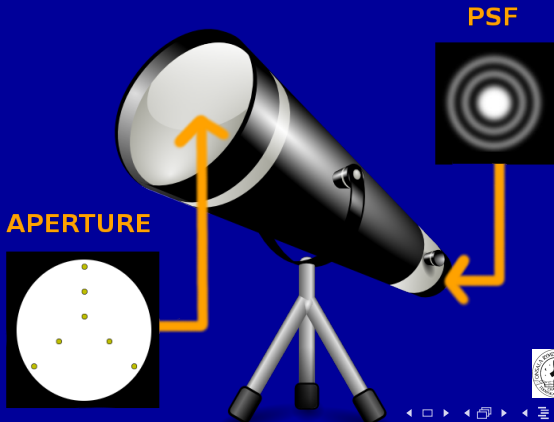
Synthesizing aperture.

The interferometer seen as a single telescope

$$\text{PSF} = \mathcal{F}(\text{APERTURE} * \text{APERTURE})$$

Aperture \rightarrow Antenna distribution! Aperture autocorr. \rightarrow UV Coverage!

PSF \rightarrow PSF of interferometer!



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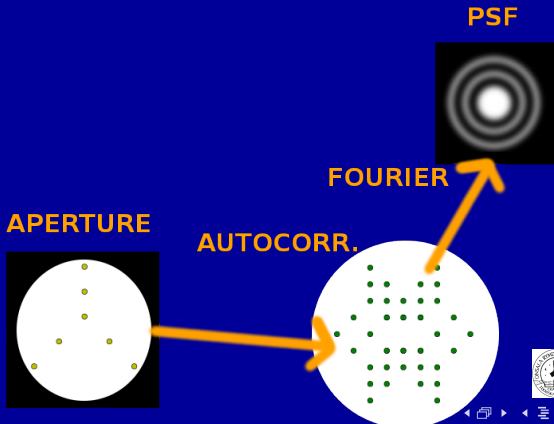
Synthesizing aperture.

The interferometer seen as a single telescope

$$\text{PSF} = \mathcal{F}(\text{APERTURE} * \text{APERTURE})$$

Aperture \rightarrow Antenna distribution! Aperture autocorr. \rightarrow UV Coverage!

PSF \rightarrow PSF of interferometer!



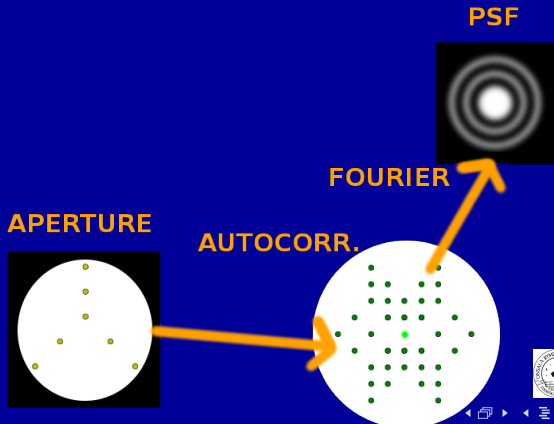
Synthesizing aperture.

The interferometer seen as a single telescope

$$\text{PSF} = \mathcal{F}(\text{APERTURE} * \text{APERTURE})$$

Aperture \rightarrow **Antenna distribution!** Aperture autocorr. \rightarrow **UV Coverage!**

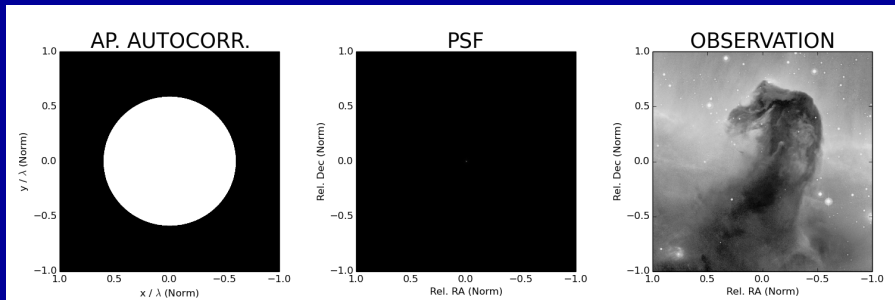
PSF \rightarrow **PSF of interferometer!** BUT BEWARE of the **zero-spacing point!**



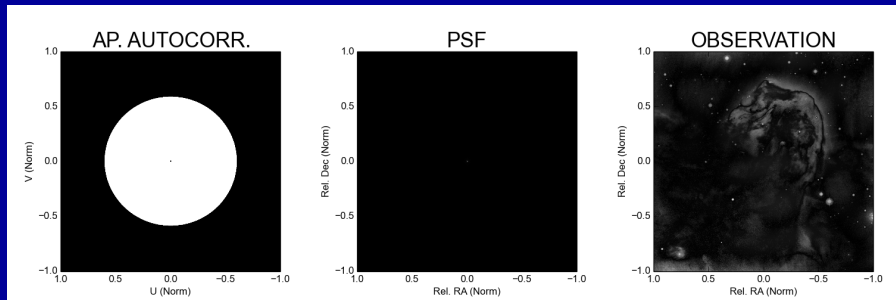
Synthesizing aperture.



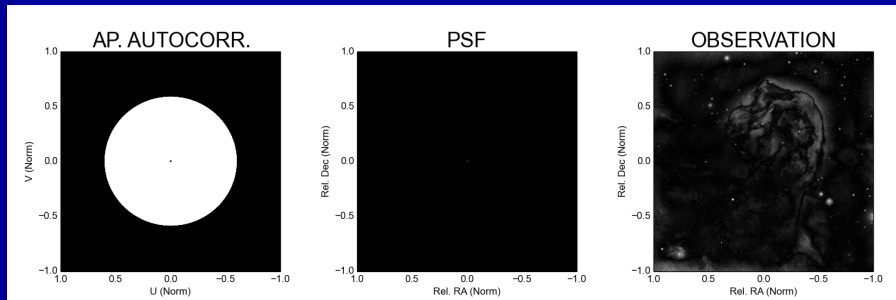
The zero-spacing hole



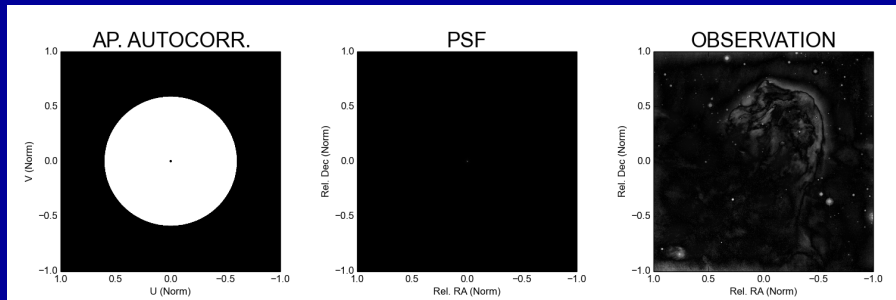
The zero-spacing hole



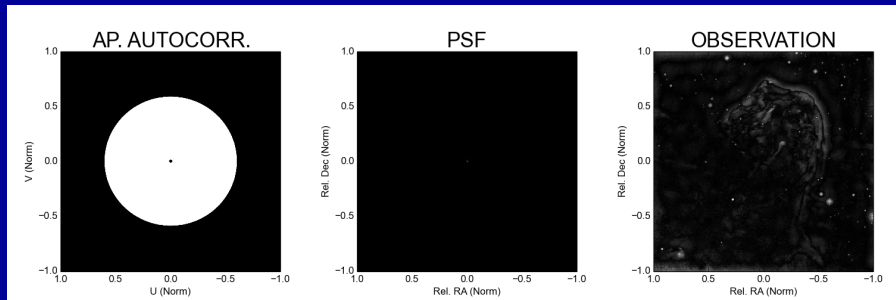
The zero-spacing hole



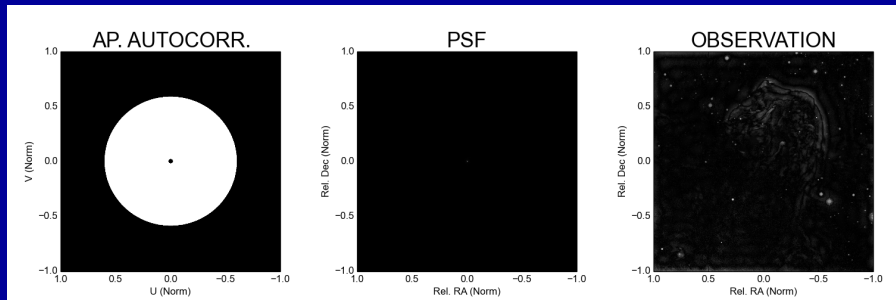
The zero-spacing hole



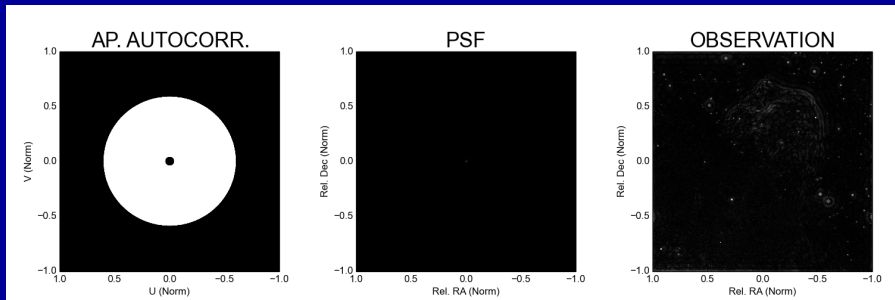
The zero-spacing hole



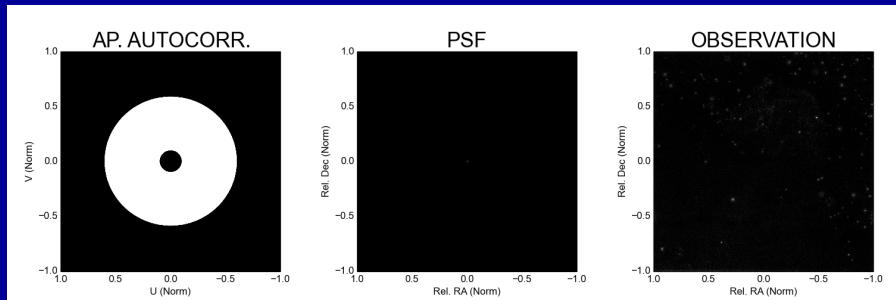
The zero-spacing hole



The zero-spacing hole



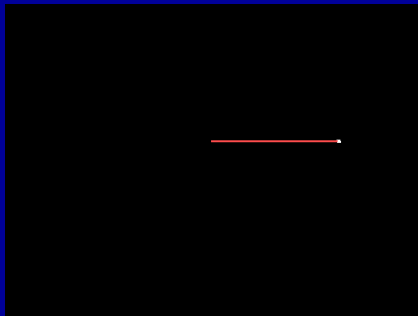
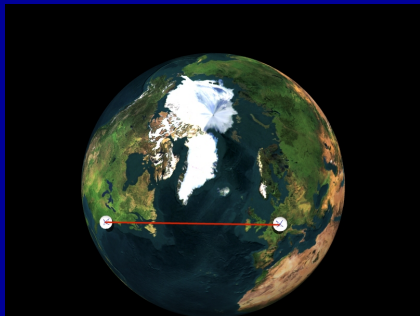
The zero-spacing hole



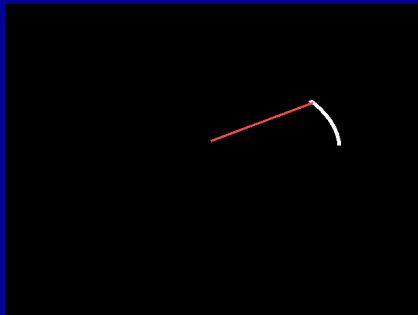
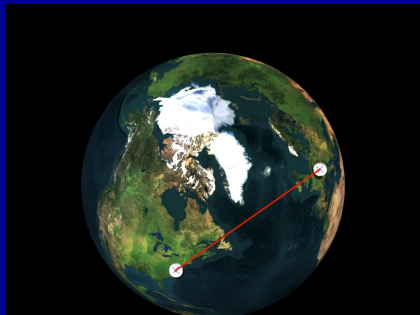
Multi-element interferometers II: Earth rotation



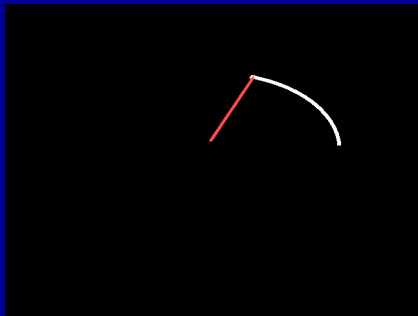
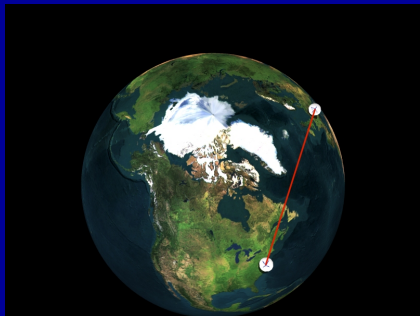
Aperture Synthesis



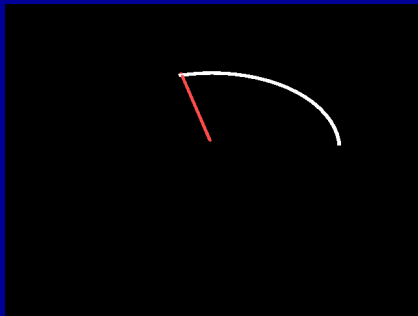
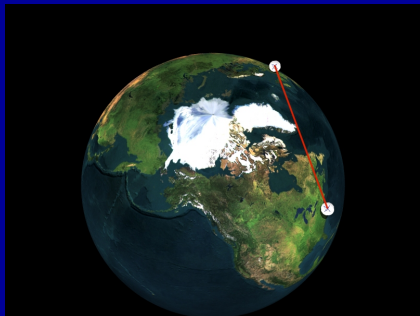
Aperture Synthesis



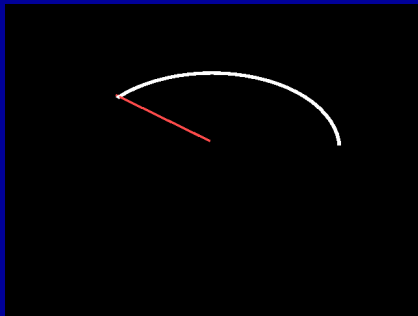
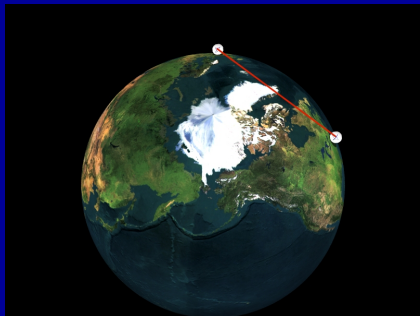
Aperture Synthesis



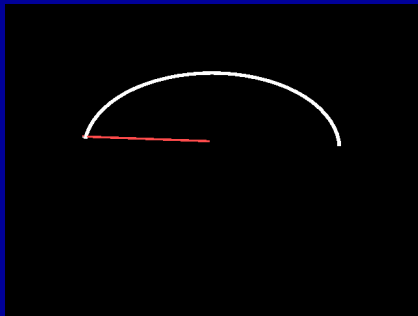
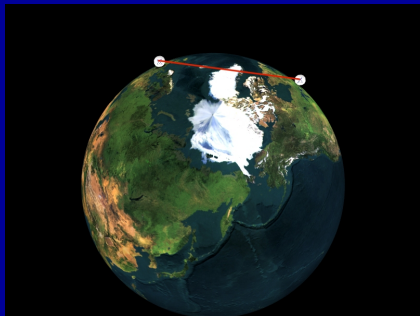
Aperture Synthesis



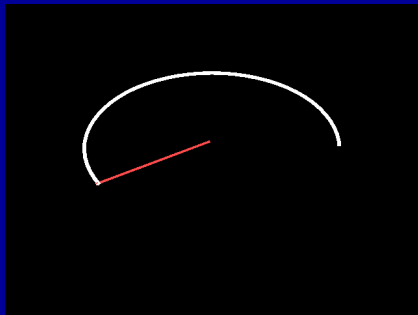
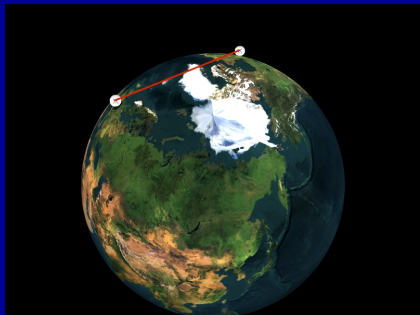
Aperture Synthesis



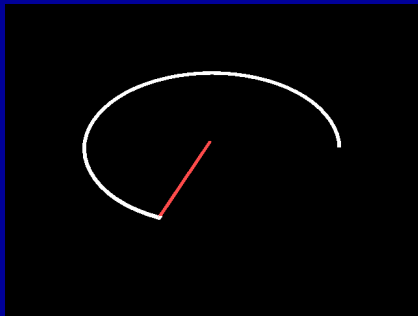
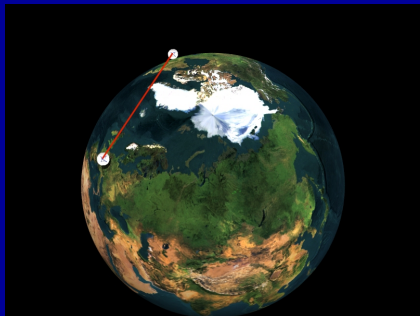
Aperture Synthesis



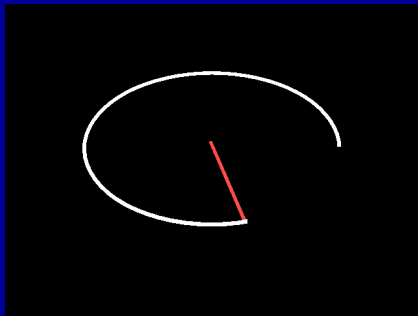
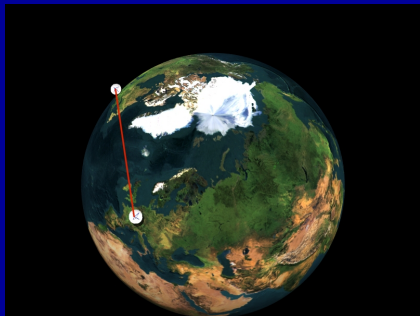
Aperture Synthesis



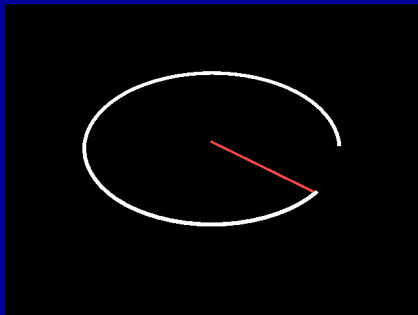
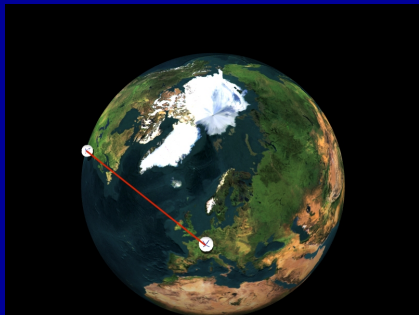
Aperture Synthesis



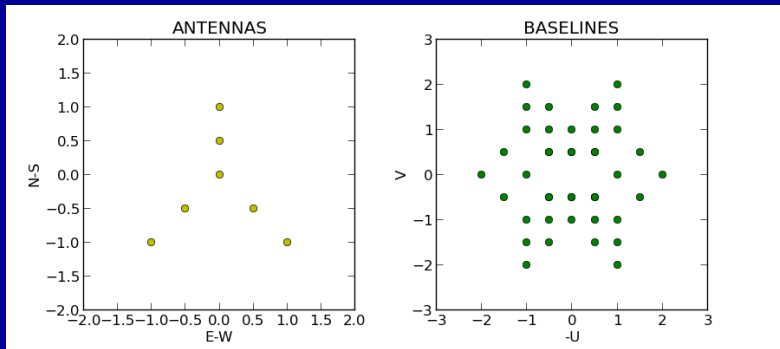
Aperture Synthesis



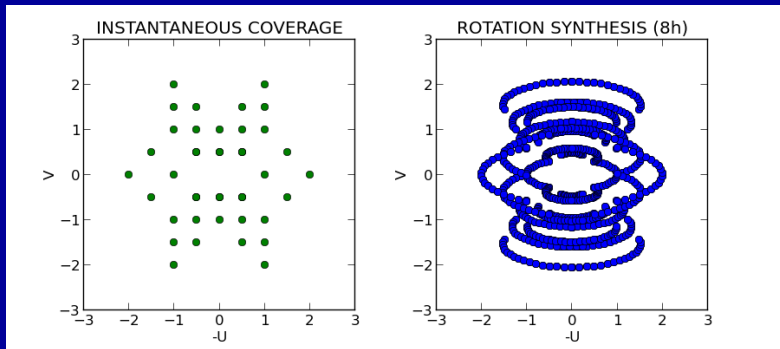
Aperture Synthesis



Aperture Synthesis



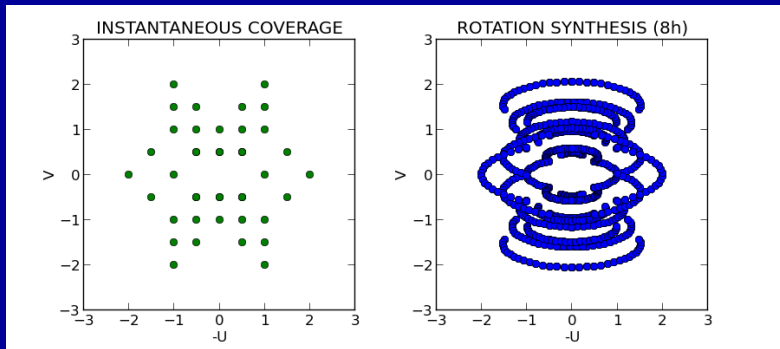
Aperture Synthesis



When the Earth rotates, the positions of the telescopes change **as seen from the source**. We can use the Earth rotation to *synthesize* UV coverage (a.k.a. aperture)!



Aperture Synthesis



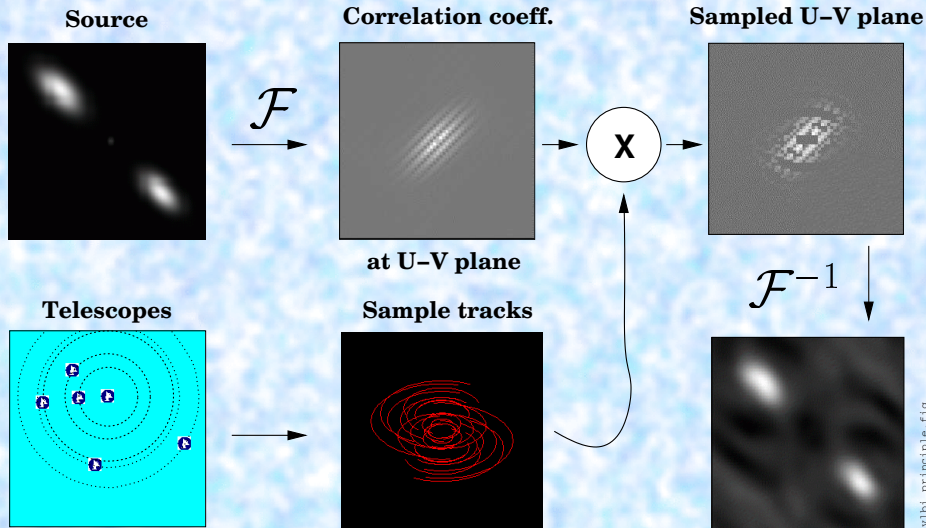
However, there will always be *holes* in the UV plane with no observations. These holes will limit the quality and fidelity of our images: contrast (a.k.a. dynamic-range) limit.

Summary

- An interferometer made of **many elements** (antennas) can be understood as a set of **many two-element** interferometers (as many as antenna pairs can be formed).
- The different antenna pairs sample **different points** of the UV plane.
- The **Earth rotation** changes the baseline projections into the source plane, hence **synthesizing** a more complete sampling of the UV plane (UV coverage).
- There are, however, **remaining holes** in the UV plane with no observations, and this will limit the quality and fidelity of our images (dynamic-range limit).
- Special care must be taken with the hole at the center of the UV plane, since it limits our sensitivity to extended structures.

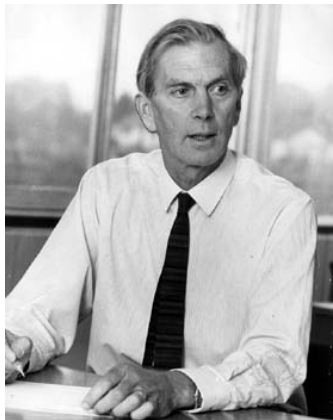


VLBI (Very Long Baseline Interferometry)



vlbi_principle.fig

How it all begun (here and there)



Sir Martin Ryle (1918 - 1984)
(G3CY)



Jorma J. Riihimaa (1933 - 2011)
(OH2PX, OH8PX)

A new radio interferometer and its application to the observation of weak radio stars

BY M. RYLE

*(Communicated by Sir Lawrence Bragg, F.R.S.—Received 19 June 1951—
Revised 10 October 1951)*

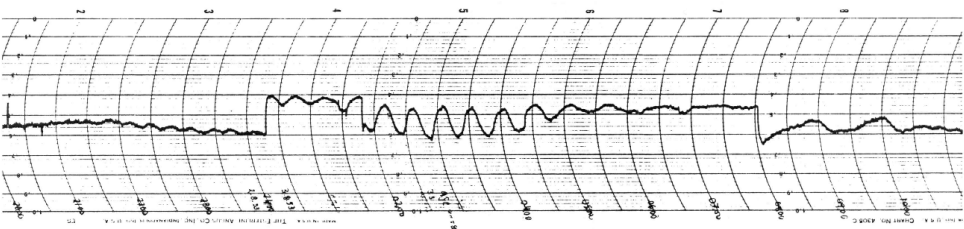
A new type of radio interferometer has been developed which has a number of important advantages over earlier systems. Its use enables the radiation from a weak 'point' source such as a radio star to be recorded independently of the radiation of much greater intensity from an extended source. It is therefore possible to use a very much greater recorder sensitivity than with earlier methods. It is, in addition, possible to use pre-amplifiers at the aeriads, and the resolving power which may be used is therefore not restricted by attenuation in the aerial cables.

Besides improved sensitivity, the new system has a number of other advantages, particularly for the accurate determination of the position of a radio source. Unlike earlier systems the accuracy of position finding is not seriously affected by rapid variations in the intensity of the radiation. It also has important applications to the measurement of the angular diameter and polarization of a weak source of radiation.

The new system has been used on wave-lengths of 1.4, 3.7, 6.7 and 8 m for the detection and accurate location of radio stars, and for the investigation of the scintillation of radio stars. It has also been used in a number of special experiments on the radiation from the sun. The results which have been obtained in these experiments have confirmed the advantages predicted analytically.



K.39. Teekkari J.J. Riihimaa etuvahvistinta virittämässä.



K.43. Ensimmäinen rekisteröinti Cass A:n ohikulusta
2-3.8.1953. (81.5 MHz)

1953

e- VLBI:

connecting remote telescopes in real-time



*Huib Jan van Langevelde
Joint Institute for VLBI in Europe
Sterrewacht Leiden*

Event Horizon Telescope (EHT)

A Global Network of Radio Telescopes



2007

2008