Signal and image processing: Interferometric Imaging

Talvikki Hovatta
Tuorla Observatory

Slides from North European Radio Astronomy School 2015 by Tuomas Savolainen (Aalto University)
Animation from Ivan Marti-Vidal (Onsala Space Observatory)

You have your calibrated visibility data. Now what?

1. **Fit simple brightness distribution models to the visibility data.**

   Pros:

   - *Works also with poorly sampled and noisy data*
   - *Visibilities have well-defined noise properties*
   - *Resolution better than Rayleigh limit achievable for high SNR data*

   Cons:

   - *Works only with simple source structures*

Examples of Visibilities – a Well Resolved Object

- The flux calibrator 3C295

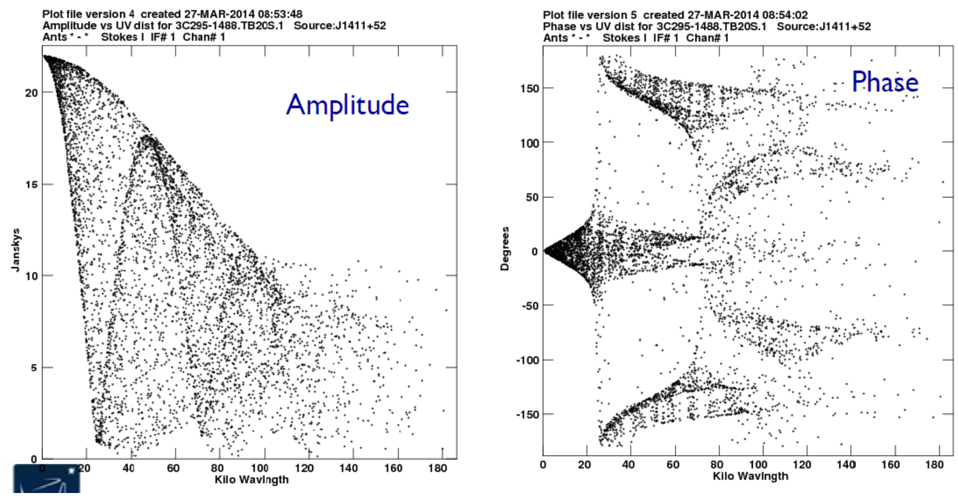

Image credit: Rick Perley

You have your calibrated visibility data. Now what?

2. **Recover an image by using inverse Fourier transform:**

$$I(l,m) = \mathcal{F}^{-1}\left(V(u,v)\right) \equiv \iint_{-\infty}^{\infty} V(u,v)e^{i2\pi(ul+vm)}dudv$$

Pros:

- *Complex structures can be studied*
- *No need to assume certain brightness distribution*

Cons:

- *Requires well-sampled visibilities*

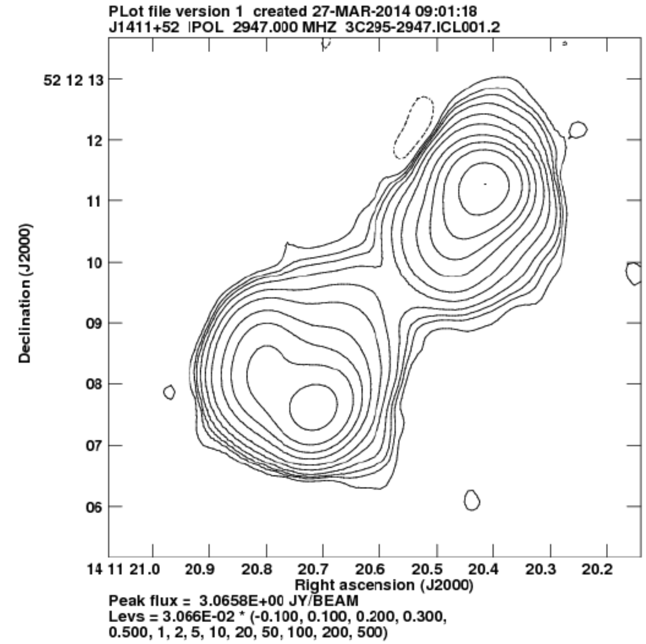

Image credit: Rick Perley

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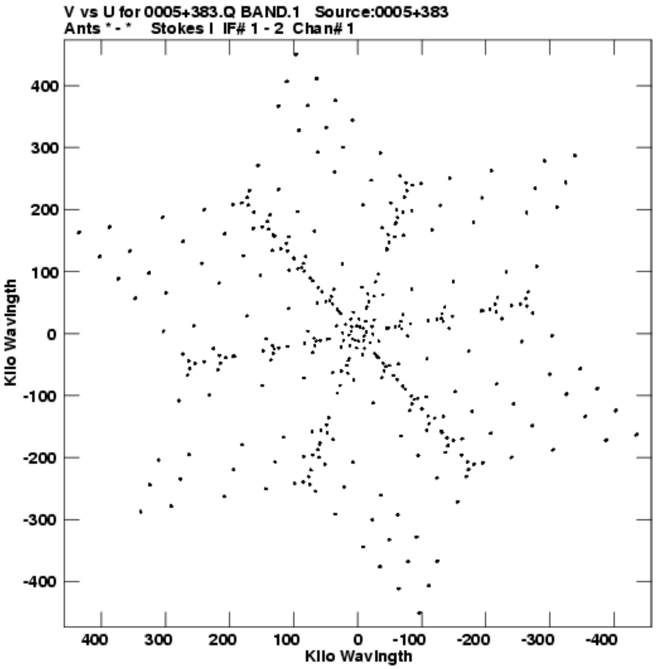
Aperture synthesis concepts

Aperture synthesis

- In principle, inverting $V(u,v) = \iint I(l,m)\, e^{-i2\pi(ul+vm)} dl dm$ gives the sky brightness distribution. This however requires measuring $V(u,v)$ everywhere in the (*u,v*) plane. Not possible!
- In reality, we aim to sample $V(u,v)$ sufficiently well in order to constrain $I(l,m)$. What is sufficiently well? Well, that is a complicated question… In any case *"(u,v)* coverage" is one of the main decisive factors between a high quality image and rubbish.
- To do well, we want:
  - Many telescopes, since the number of instantaneous (*u,v*) samples is N(N-1), where N is the number of telescopes
  - Long synthesis time for changing baseline projections as Earth rotates. However, be careful if the source is variable!

Examples of (*u*,*v*) plane sampling

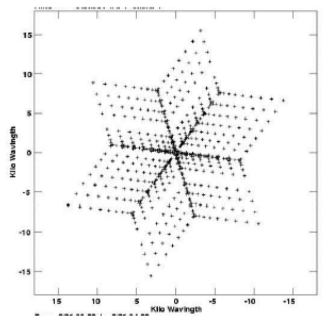


Visibility sampling for a VLA snapshot

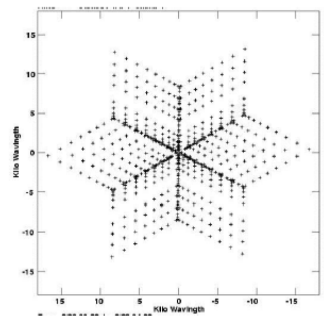
Examples of (*u,v*) plane sampling

Sample VLA (U,V) plots for 3C147 (δ = 50)

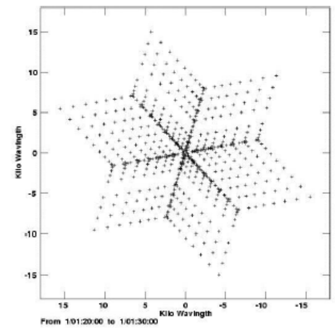
- Snapshot (u,v) coverage for HA = -2, 0, +2 (with 26 antennas).



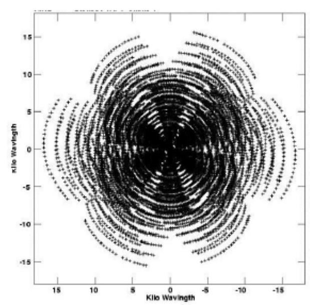
HA = -2h



HA = 0h



HA = 2h

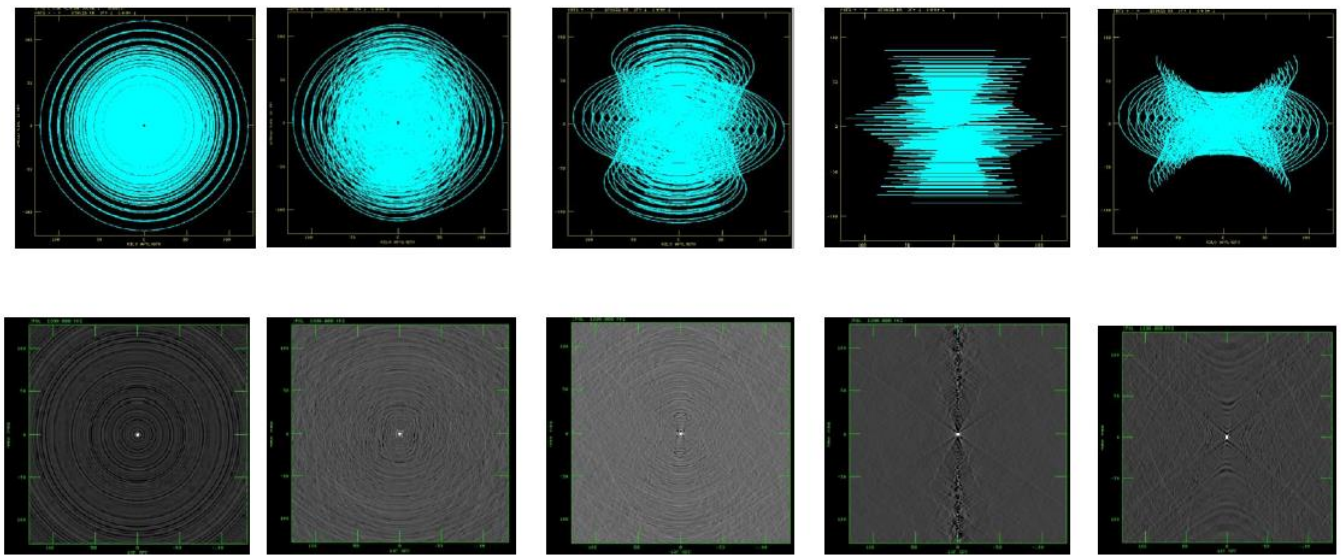


Coverage over all four hours.

Image credit: Rick Perley

Examples of (*u*,*v*) plane sampling

VLA Coverage and Beams

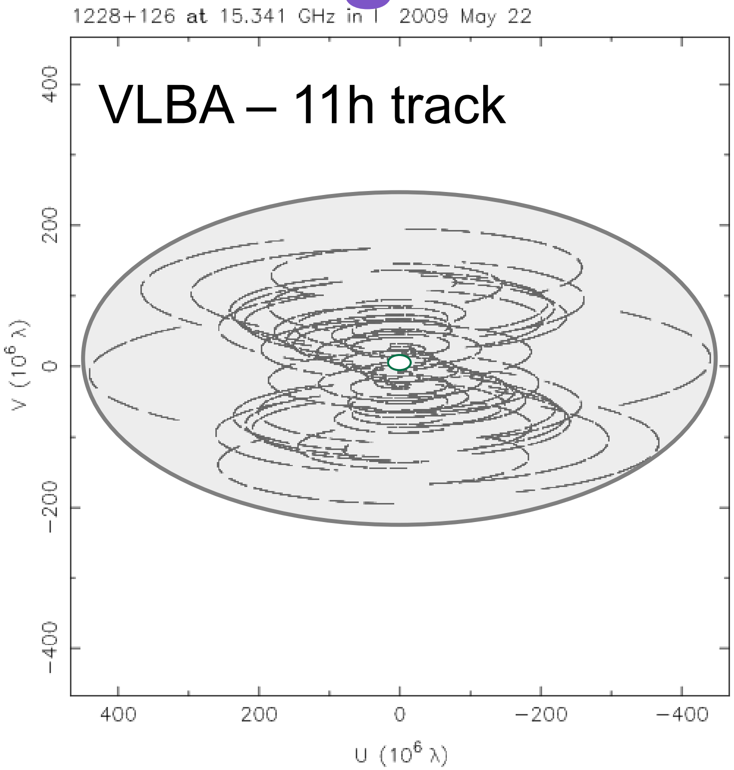


δ=90 δ=60 δ=30 δ=0 δ=-30

Sources at different declinations

Image credit: Rick Perley

What does (*u,v*) coverage mean to your image?



- Outer boundary limits the angular resolution
- Inner boundary limits the sensitivity to large-scale emission structure
- Imperfect sampling in-between limits the image fidelity – there is information missing!

Formal description of a discrete sampling of the (u,v) plane

Visibility plane is sampled at discrete points given by sampling function:

$$S(u,v) = \sum_k \delta(u-u_k)\delta(v-v_k)$$

If we take an inverse FT of the sampled visibility function, we get a "dirty" image:

$$I^D(l,m) = \mathcal{F}^{-1}(S(u,v)V(u,v))$$

Convolution theorem says:

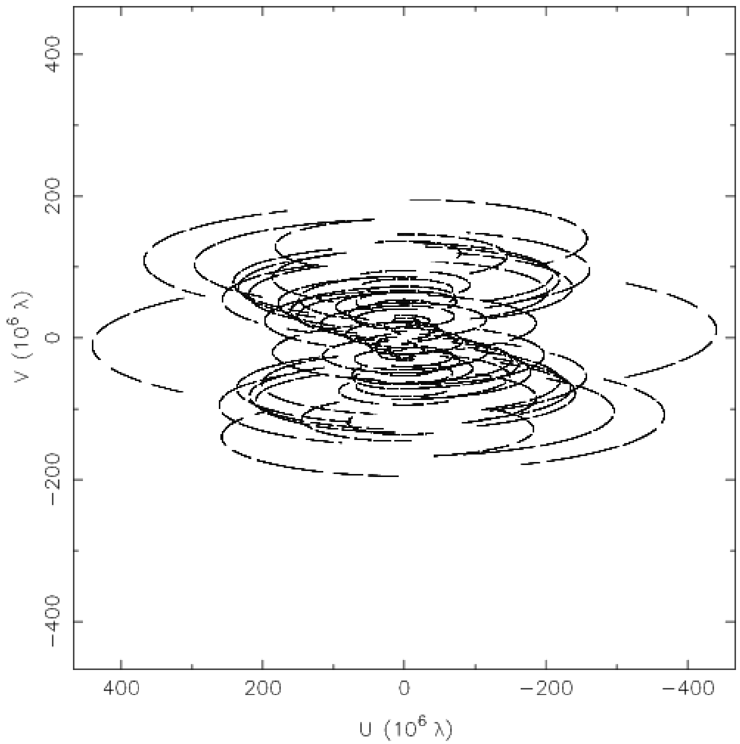
$$I^D(l,m) = b(l,m) * I(l,m)$$

So, $I^D(l,m)$ is a convolution of the true sky brightness distribution and the interferometer beam:

$$b(l,m) = \mathcal{F}^{-1}(S(u,v))$$

Interferometer beam

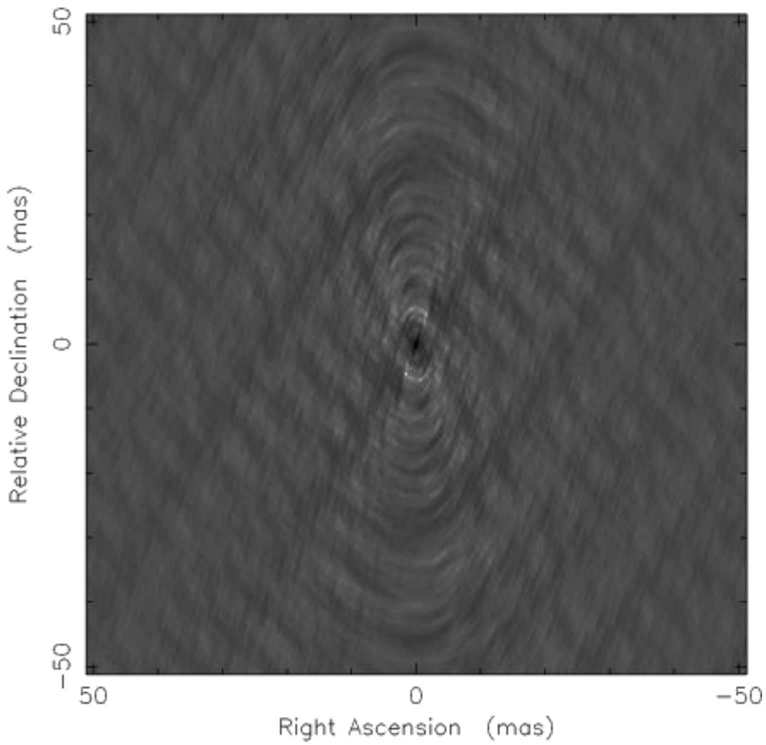
(u,v) plane sampling



$\mathcal{F}$

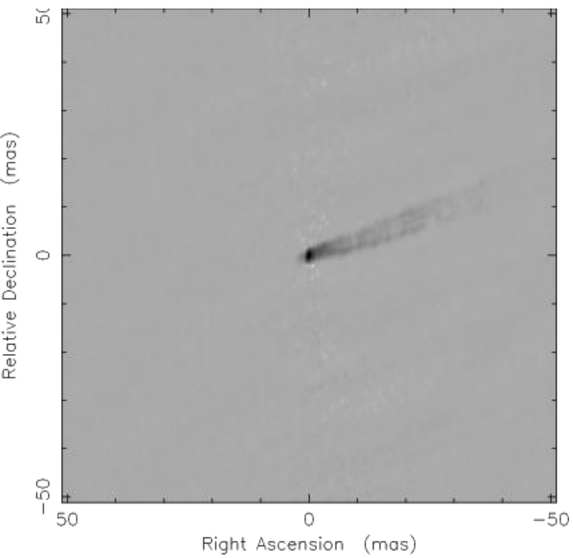
⇋

Interferometer beam



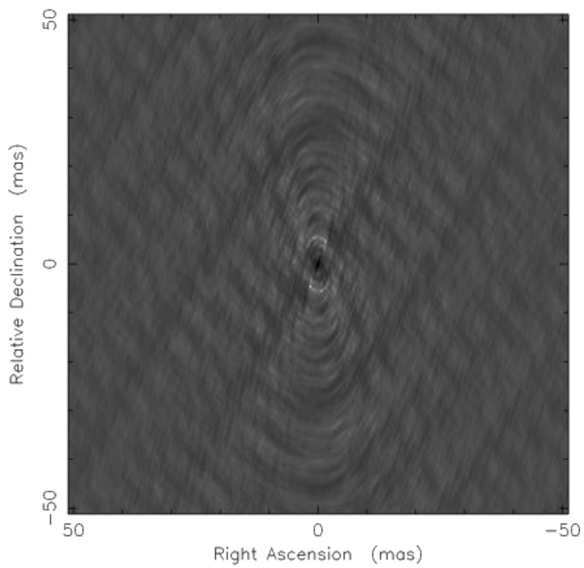
Dirty image

source structure



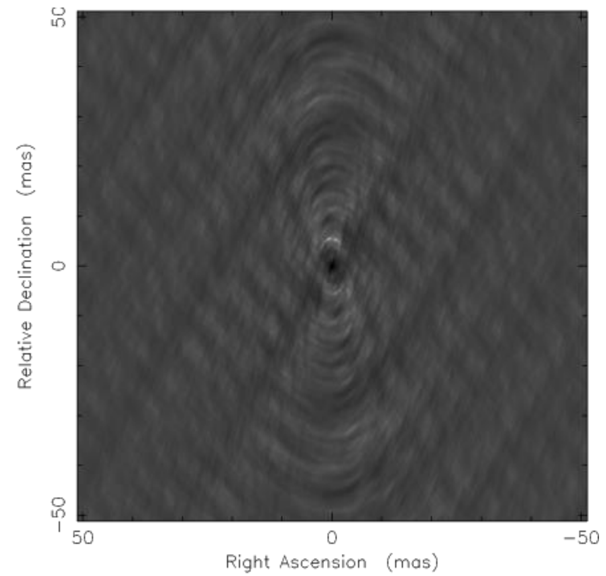
*

interferometer beam

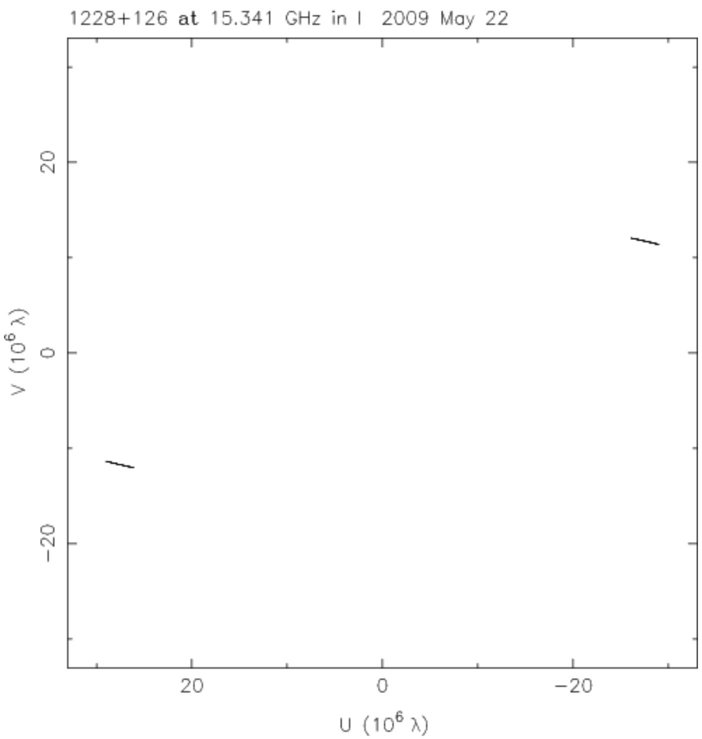


=

dirty image



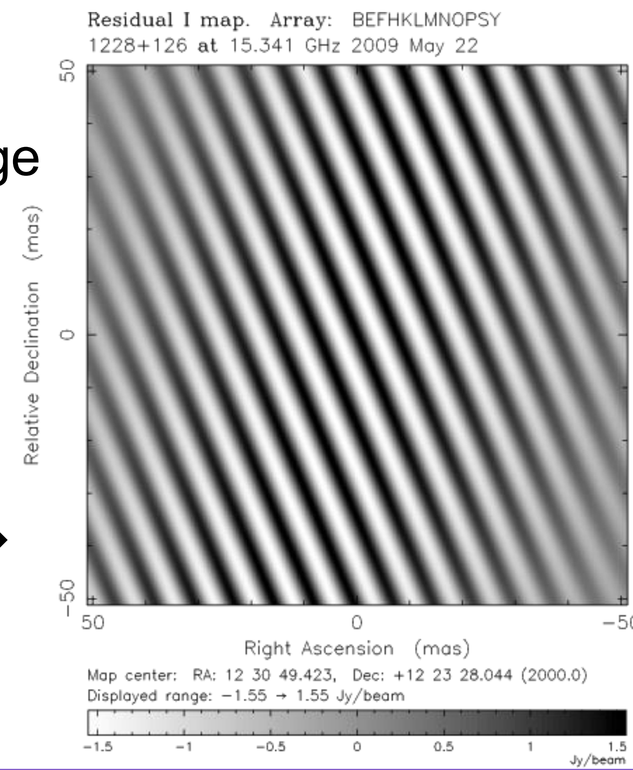
Example: Beam shape with increasing number of (u,v) samples



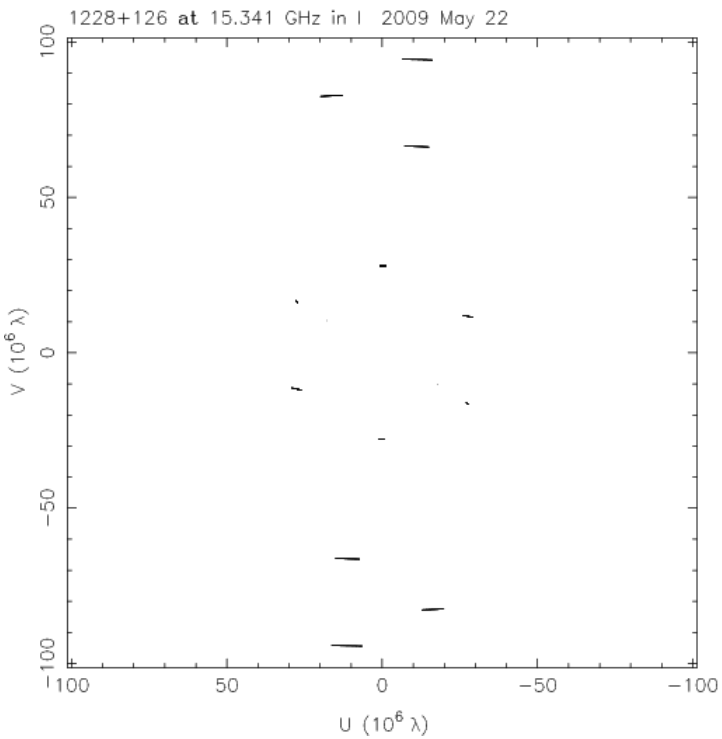
←(u,v) coverage

2 stations
25 min

Dirty image →



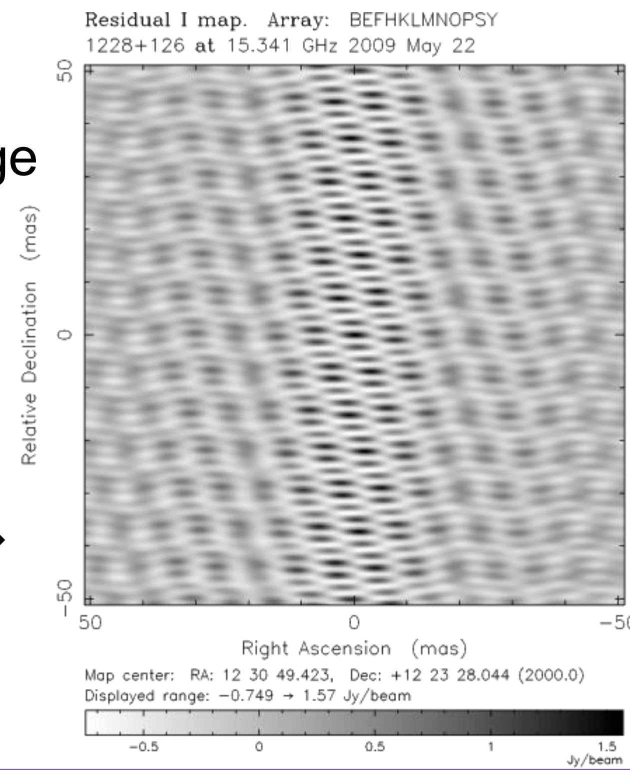
Example: Beam shape with increasing number of (u,v) samples



←(u,v) coverage

4 stations
25 min

Dirty image →



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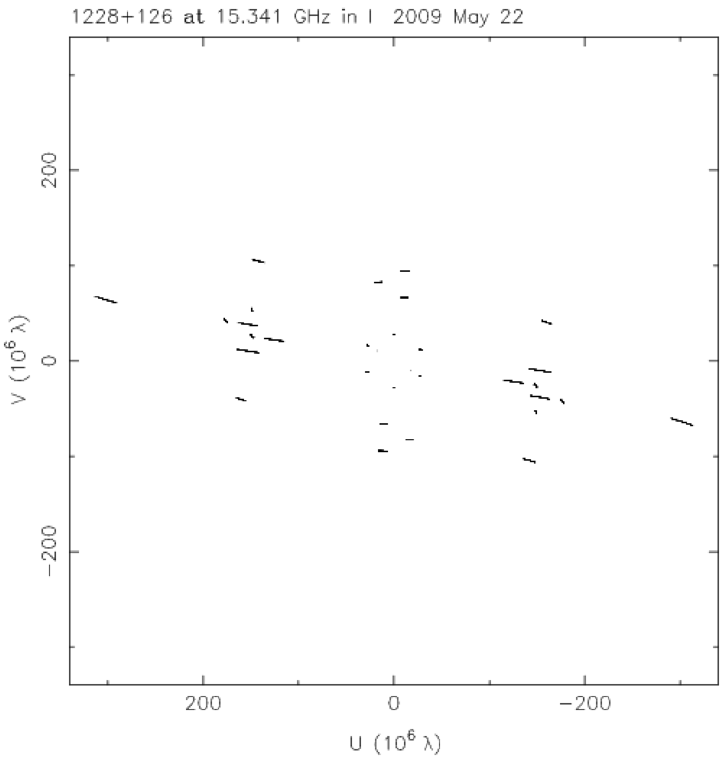
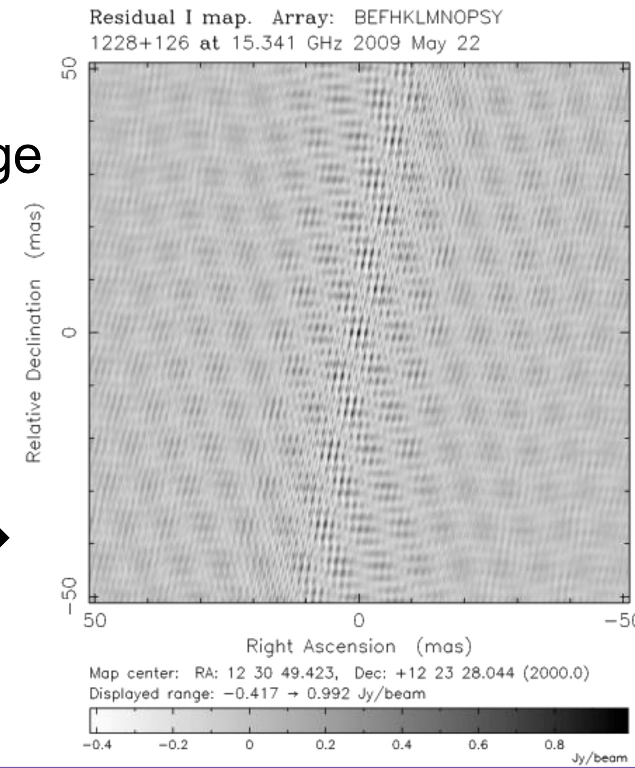
Example: Beam shape with increasing number of (u,v) samples



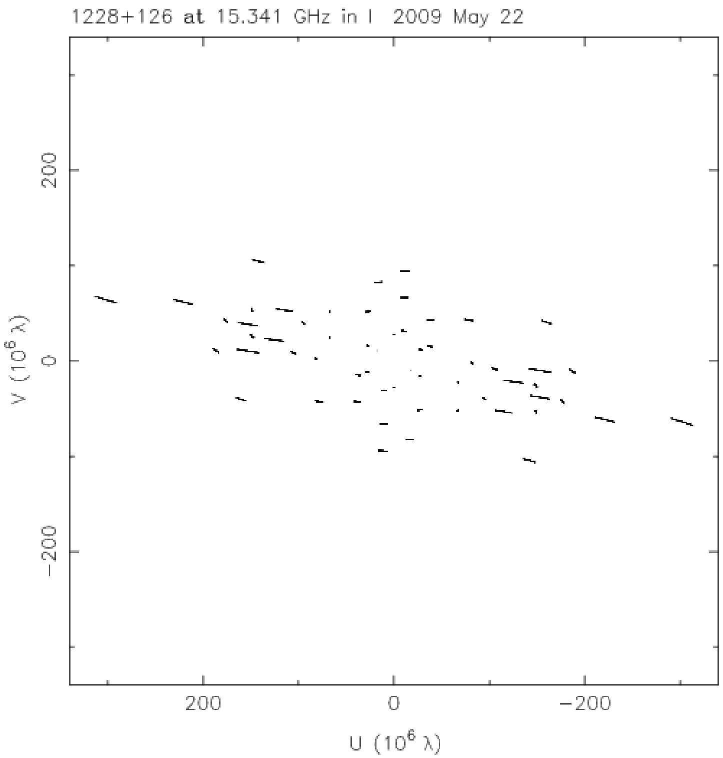
←(u,v) coverage

6 stations
25 min

Dirty image →



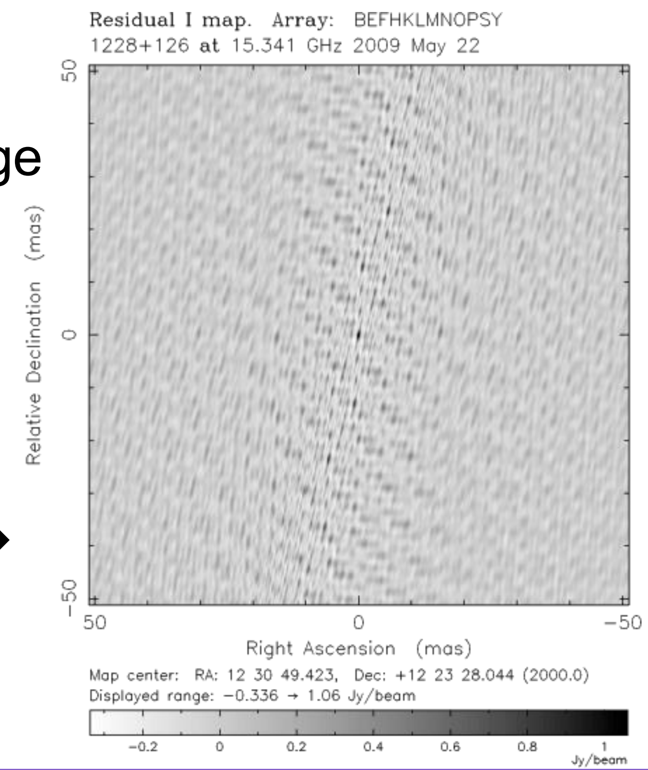
Example: Beam shape with increasing number of (u,v) samples



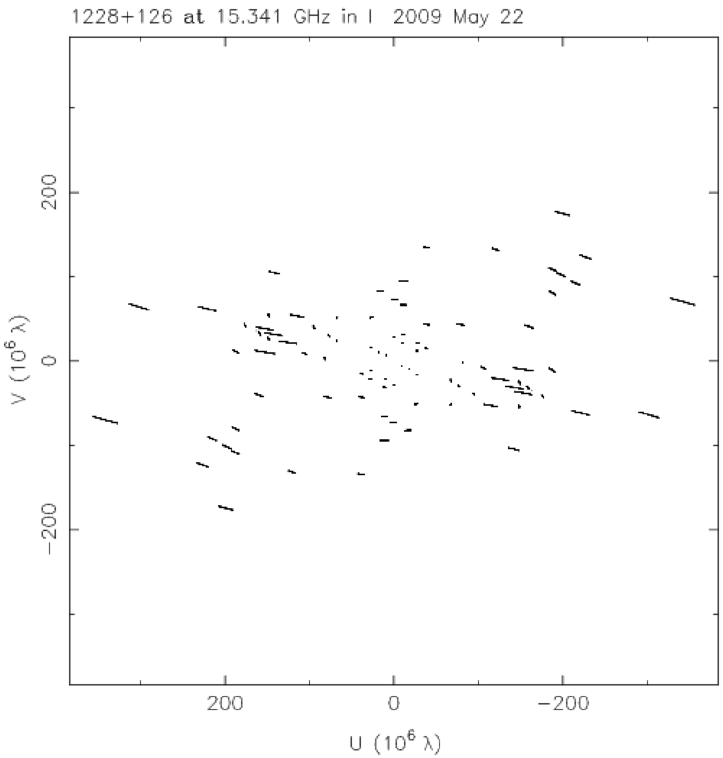
←(u,v) coverage

8 stations
25 min

Dirty image →



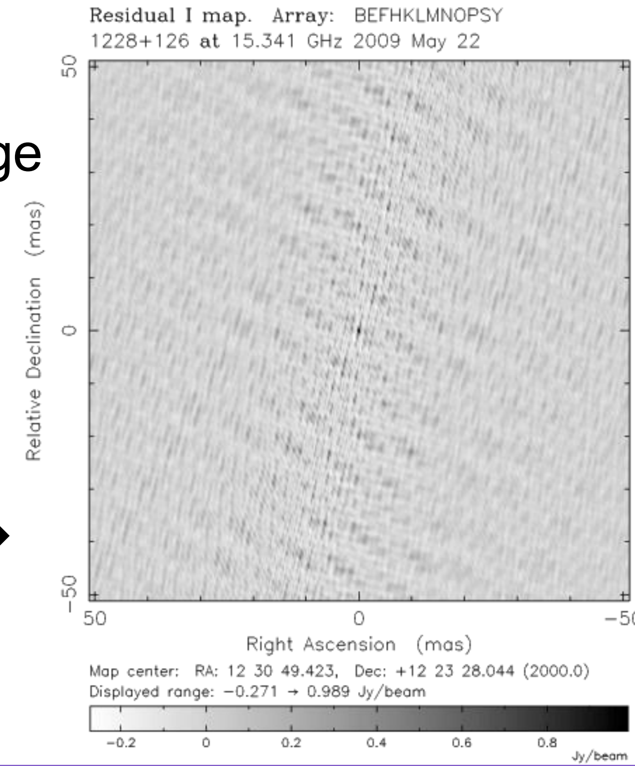
Example: Beam shape with increasing number of (u,v) samples



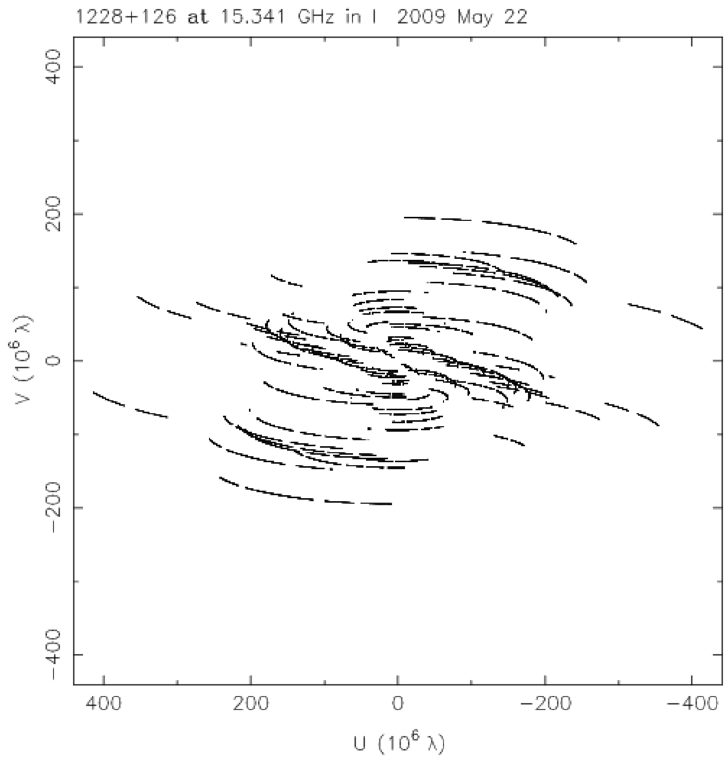
←(u,v) coverage

10 stations
25 min

Dirty image →



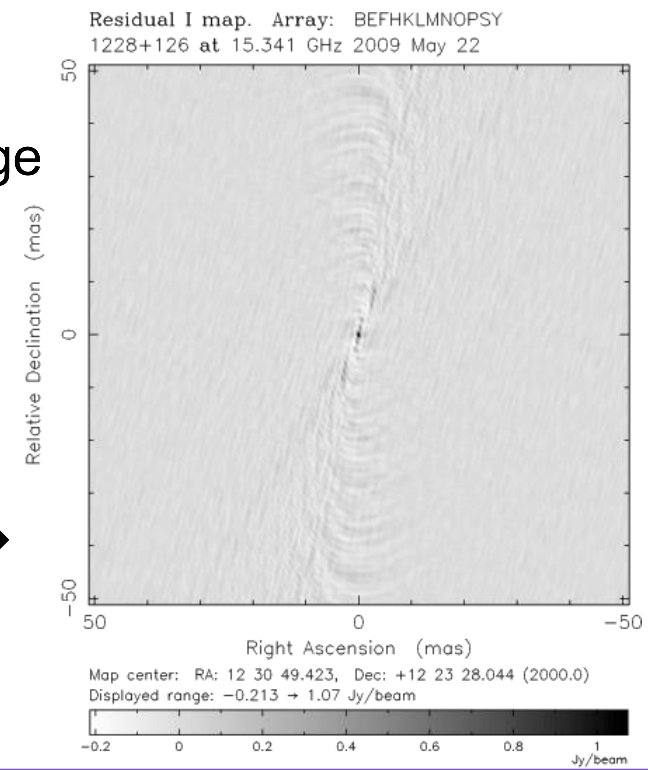
Example: Beam shape with increasing number of (u,v) samples



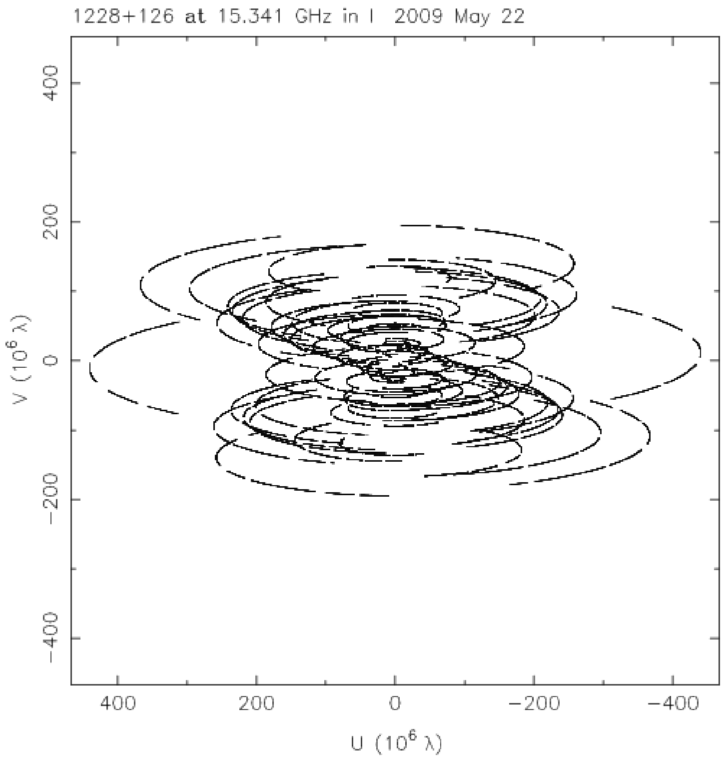
←(u,v) coverage

10 stations
5 hours

Dirty image →



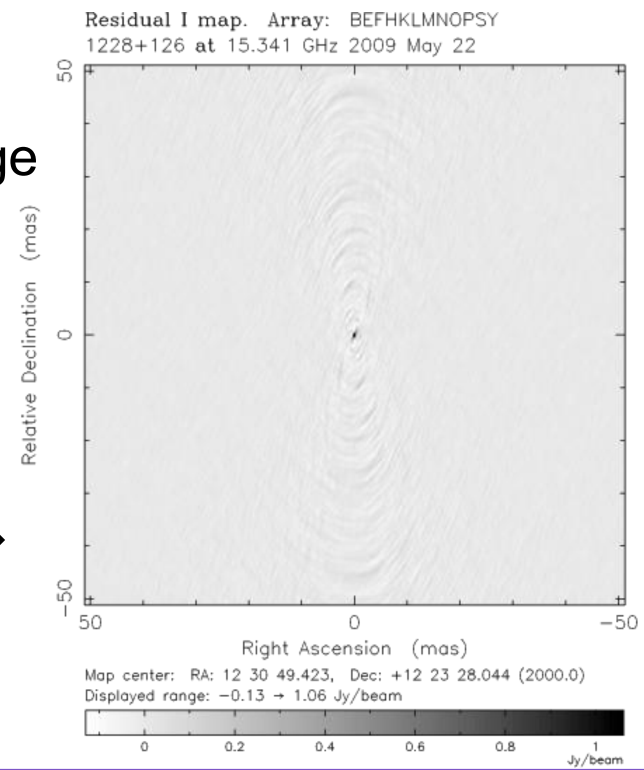
Example: Beam shape with increasing number of (u,v) samples

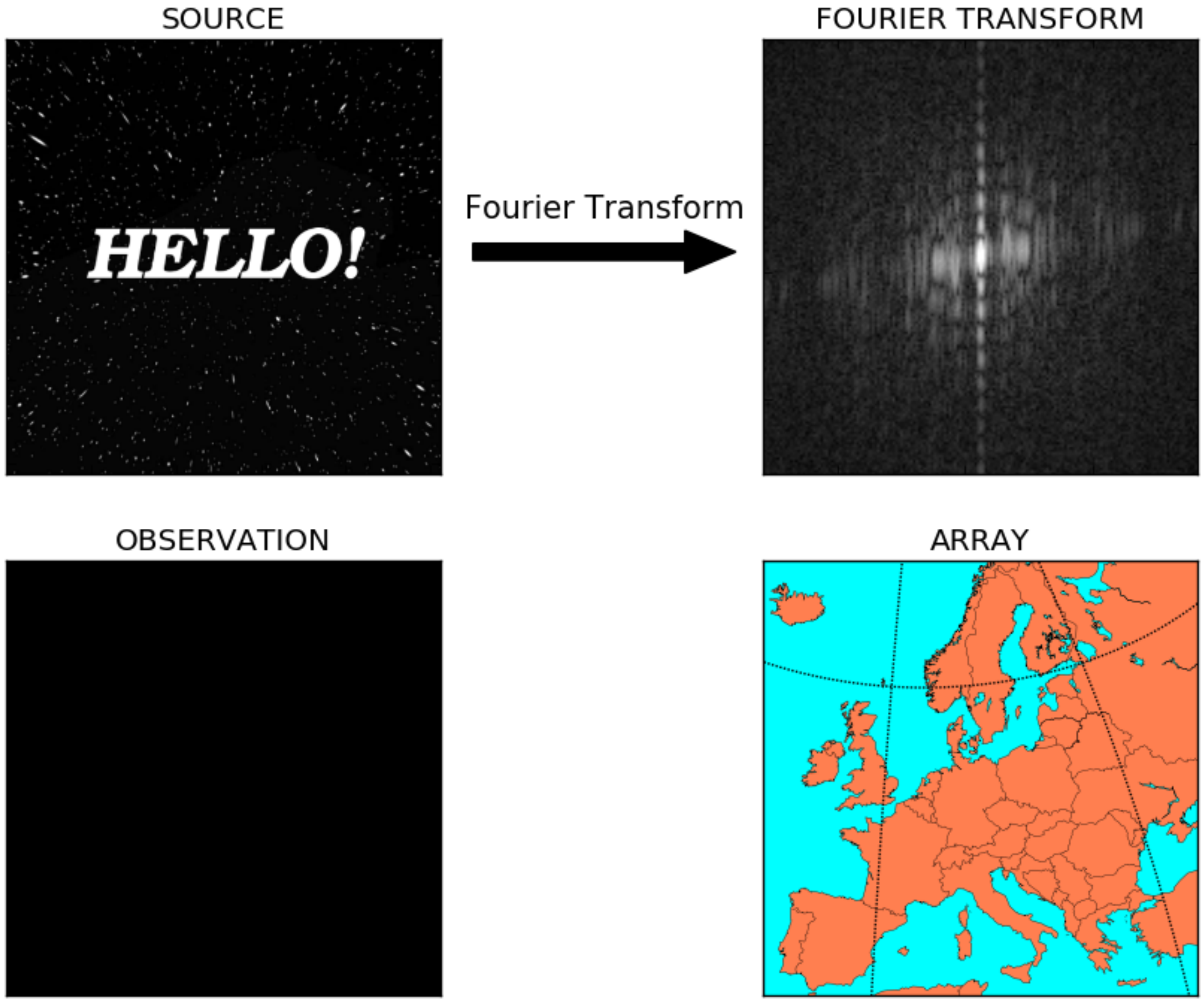


←(u,v) coverage

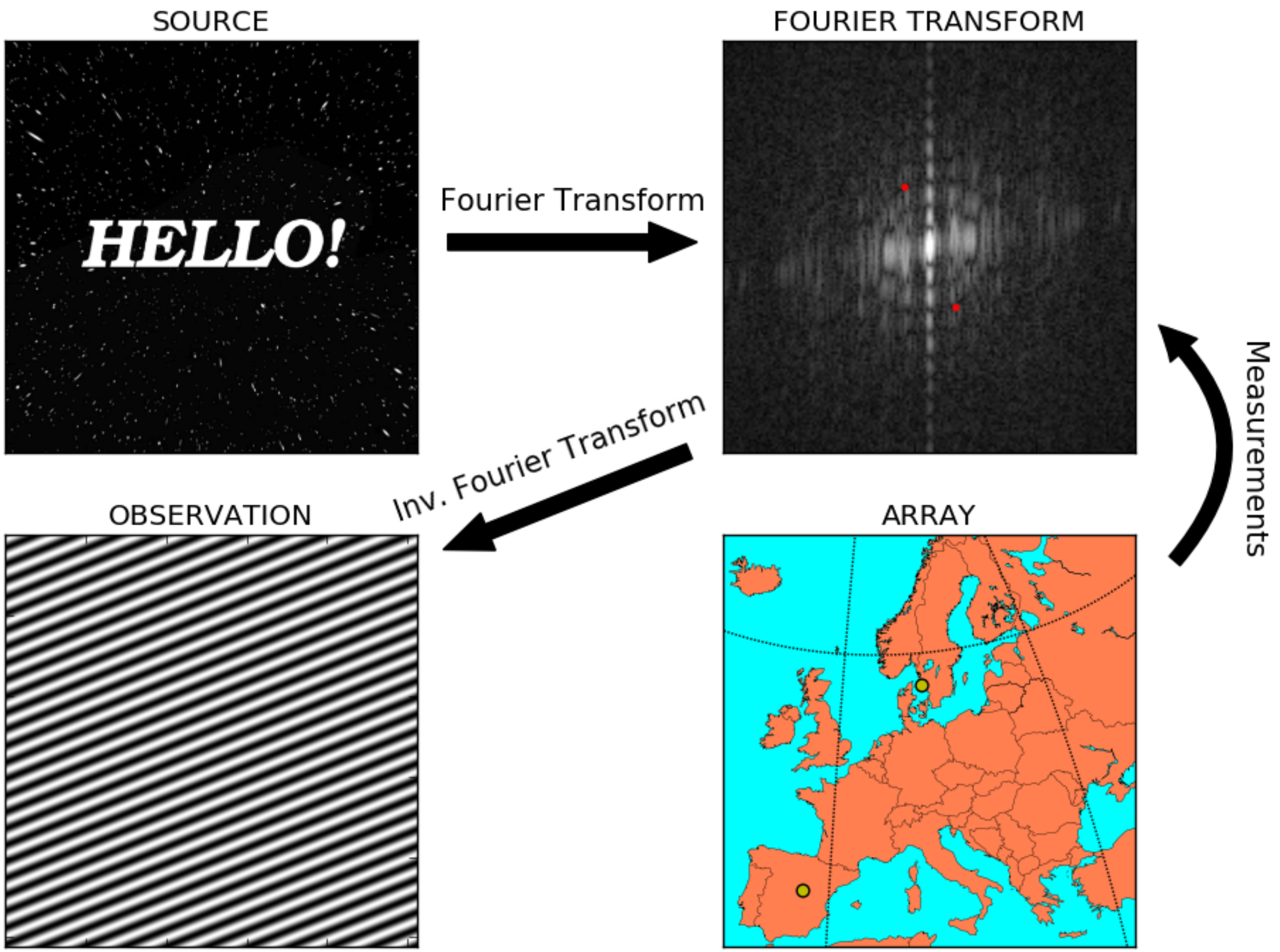
10 stations
11 hours

Dirty image →

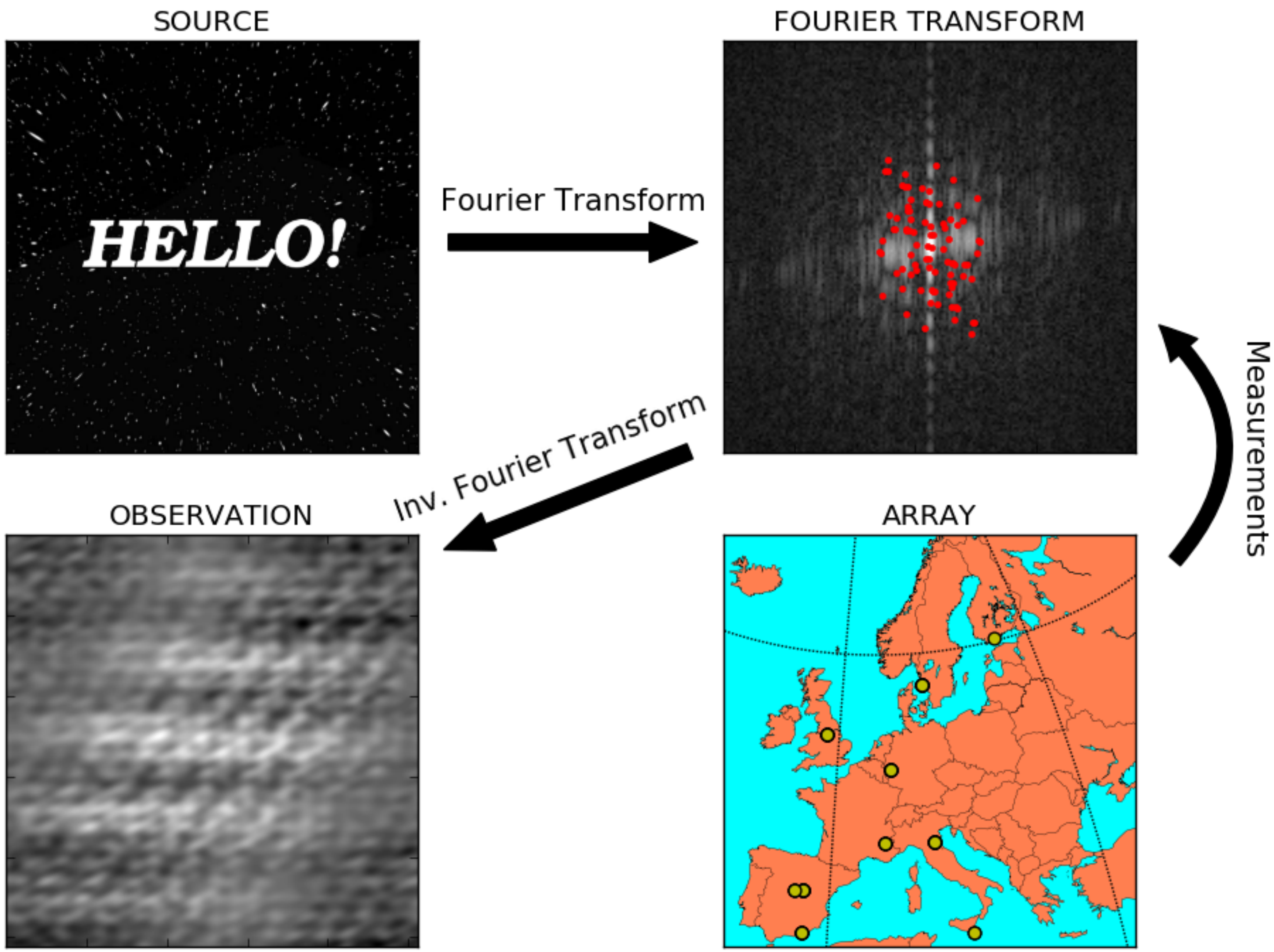




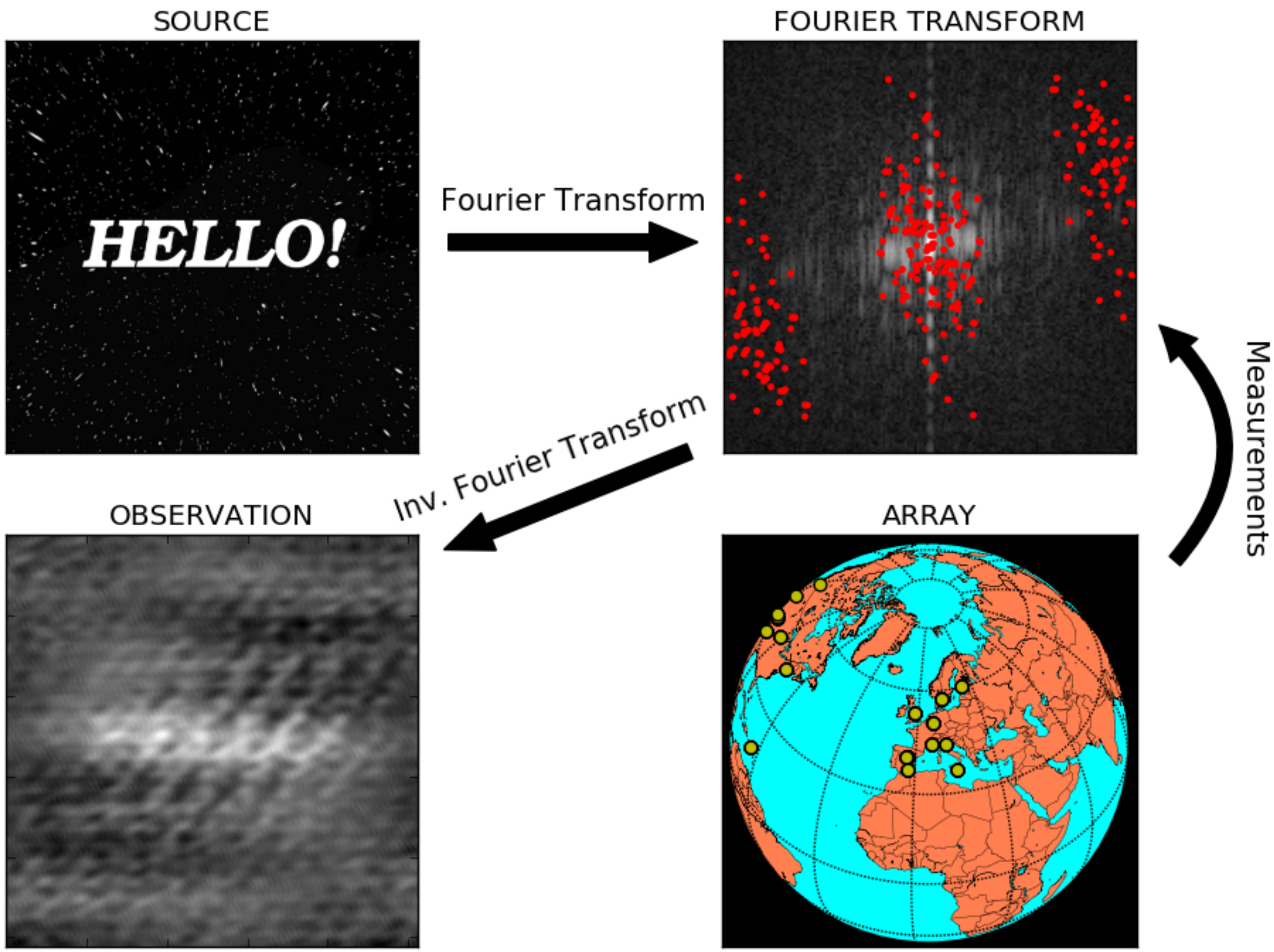
Courtesy of Ivan Marti-Vidal



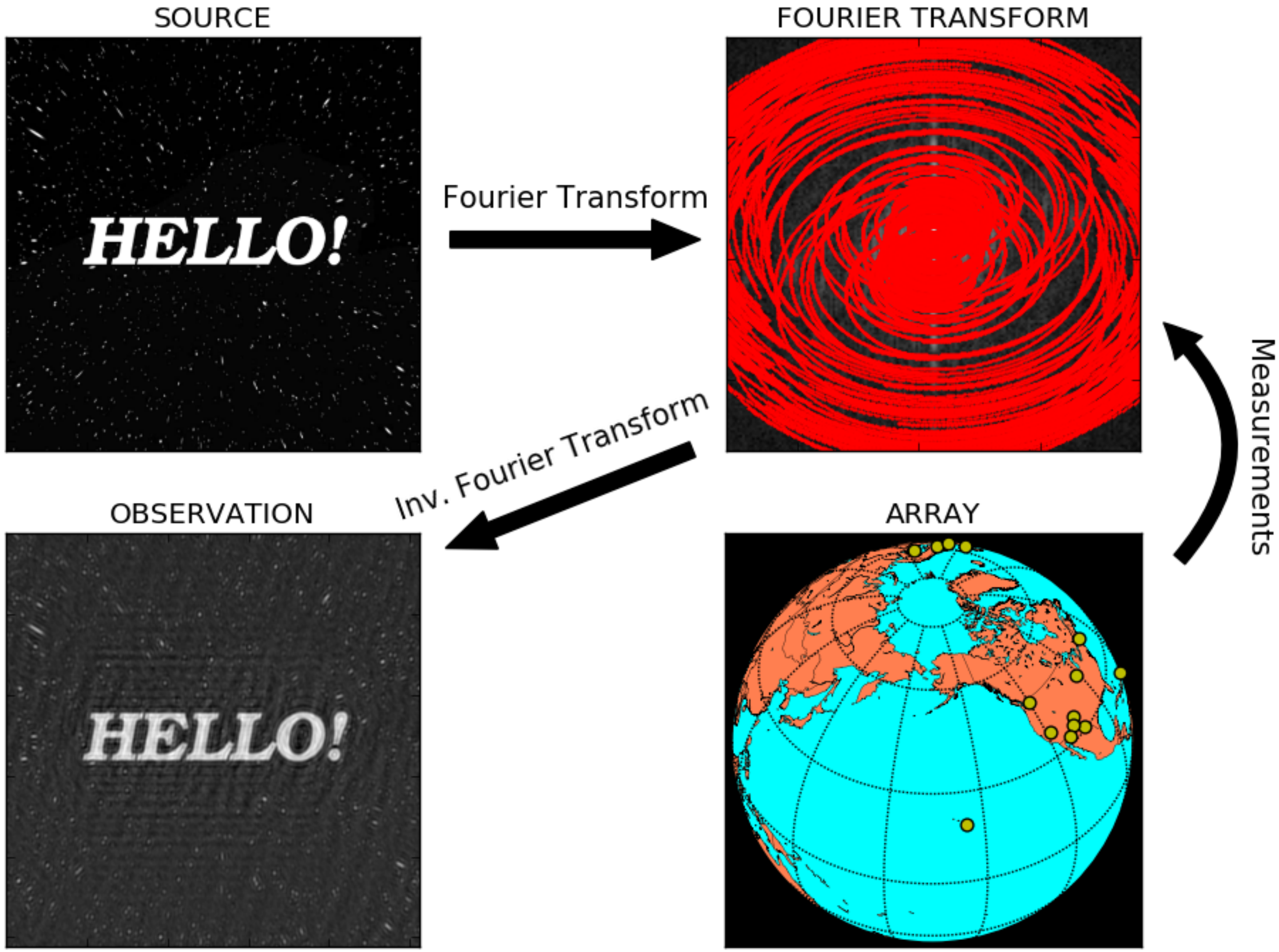
Courtesy of Ivan Marti-Vidal



Courtesy of Ivan Marti-Vidal



Courtesy of Ivan Marti-Vidal



Courtesy of Ivan Marti-Vidal

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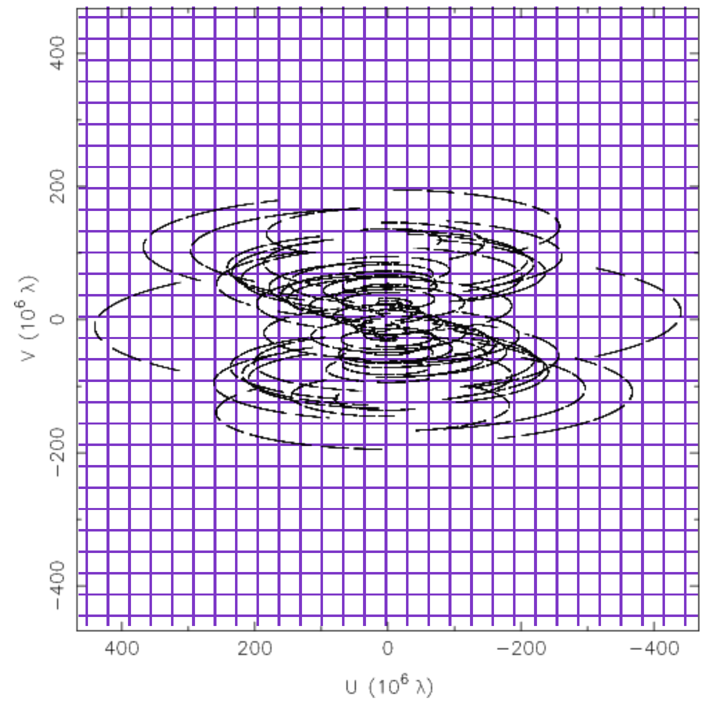
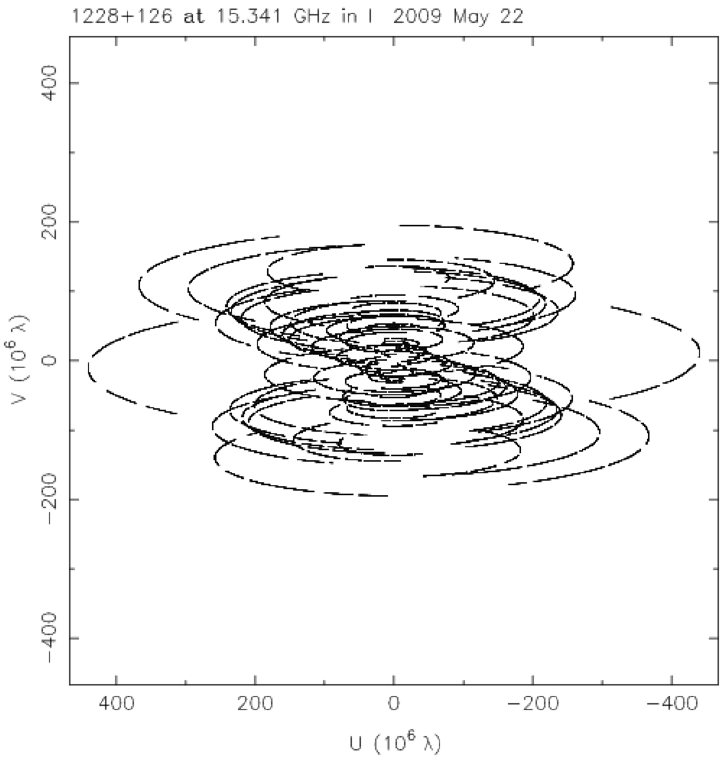
Imaging process in practice

A note about practical Fourier transformation

- Fast Fourier Transform (FFT) is typically used to invert the data, since it is much faster than direct FT ($\mathcal{O}(N^2 \log_2 N)$ vs. $\mathcal{O}(N^4)$) for an image of $\mathrm{N} \times N$ pixels and $\sim N^2$data points
- FFT requires data points on a rectangular grid → *V*(*u*,*v*) needs to be interpolated and resampled for FFT



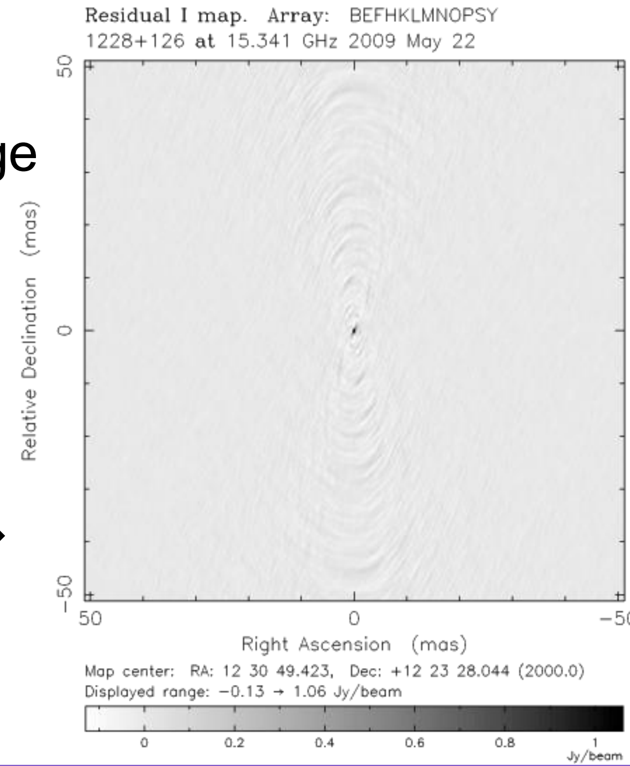
Remember this? Despite 11 hours with 10 antennas, the dirty image is useless!



←(u,v) coverage

10 stations
11h track

Dirty image →



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Going beyond the dirty image – deconvolution

- There exists an infinite number of solutions $I^s(l,m)$ that satisfy $I^D(l,m) = b(l,m) * I(l,m)$. This is because there exist functions Z with $Z * B = 0$. Therefore, if $I^s(l,m)$ is a solution, so is $I^s(l,m) + \alpha Z(l,m)$, if no extra constraints exist. Traditional linear deconvolution methods do not work!
- Typically one uses non-linear deconvolution algorithms to interpolate and extrapolate the part of the visibility function that was not measured.
- These methods require some form of regularization. This means that we need some *a priori* assumptions about the source structure in order to recover it. Luckily, quite simple assumptions suffice: 1) finite source size, 2) positivity of the true brightness distribution, 3) smoothness of the true brightness distribution.

Deconvolution with CLEAN algorithm

CLEAN is the most widely used algorithm (implementations in CASA, AIPS, Difmap …)

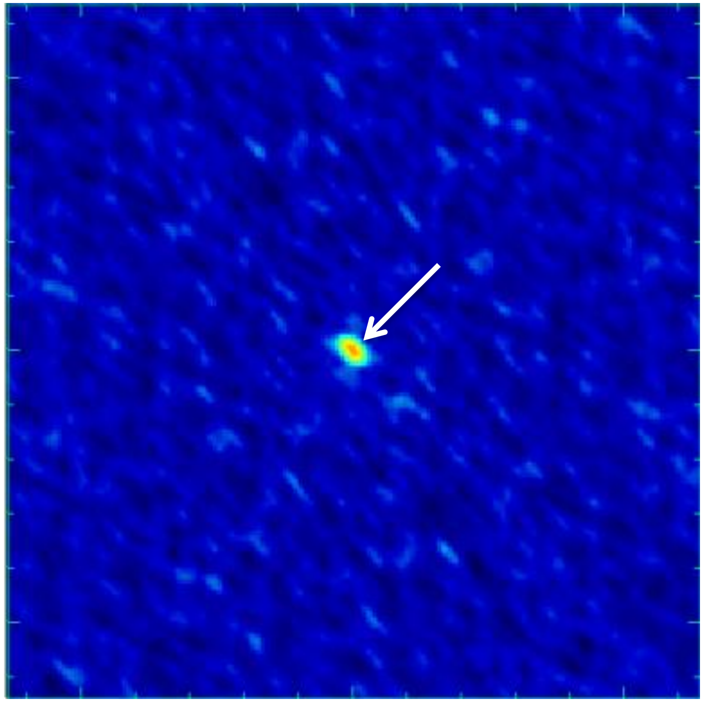
- Fits and subtracts the interferometer beam iteratively
- Original version by Högbom (1974), several improvements later
- Assumes that source structure can be presented as a sum of a finite number of point sources
- User can supply *a priori* information by restricting the area at which CLEAN is allowed to work ("CLEAN windows")
- Has problems with diffuse emission (creates "spotty" structures)
- Instabilities: striping around extended sources is a common artefact

Deconvolution with CLEAN algorithm

Basic algorithm:

Initialize: *residual map* = *dirty map* and list of δ-components = empty

1. Find the peak in the residual map, identify it as a point source

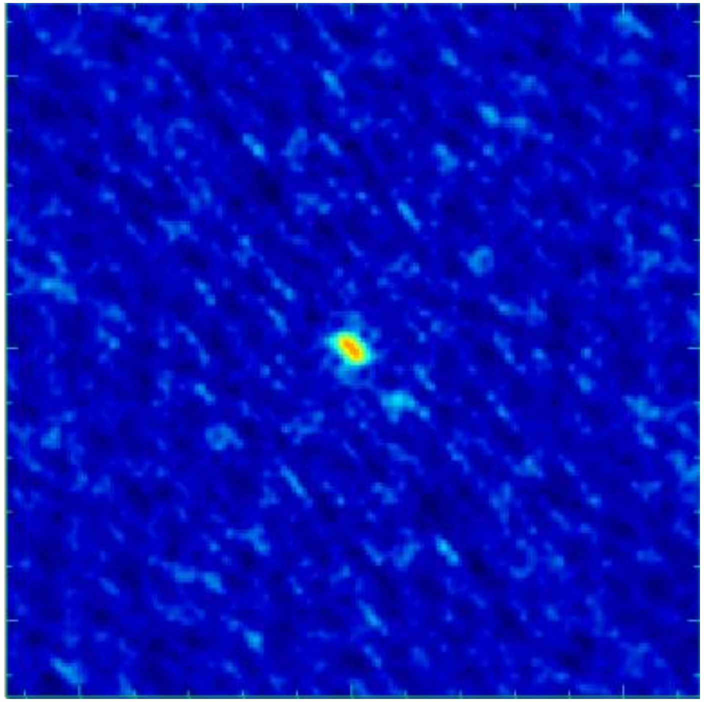


Deconvolution with CLEAN algorithm

Basic algorithm:

Initialize: *residual map* = *dirty map* and list of δ-components = empty

1. Find the peak in the residual map, identify it as a point source
2. Subtract this point source, scaled by *loop_gain* and convolved with the interferometer beam, from the residual image



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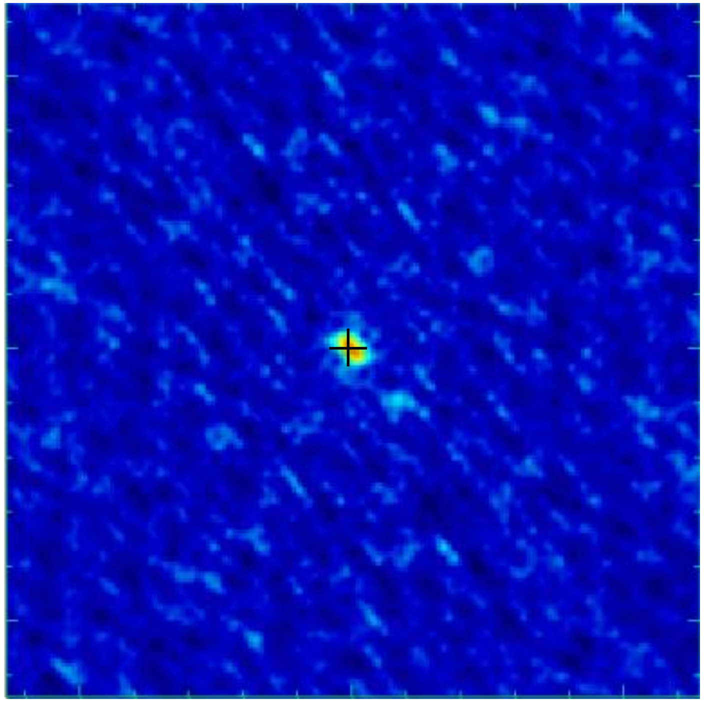
1. Find the peak in the residual map, identify it as a point source
2. Subtract this point source, scaled by *loop_gain* and convolved with the interferometer beam, from the residual image
3. Save the position and subtracted flux to the list of δ-components

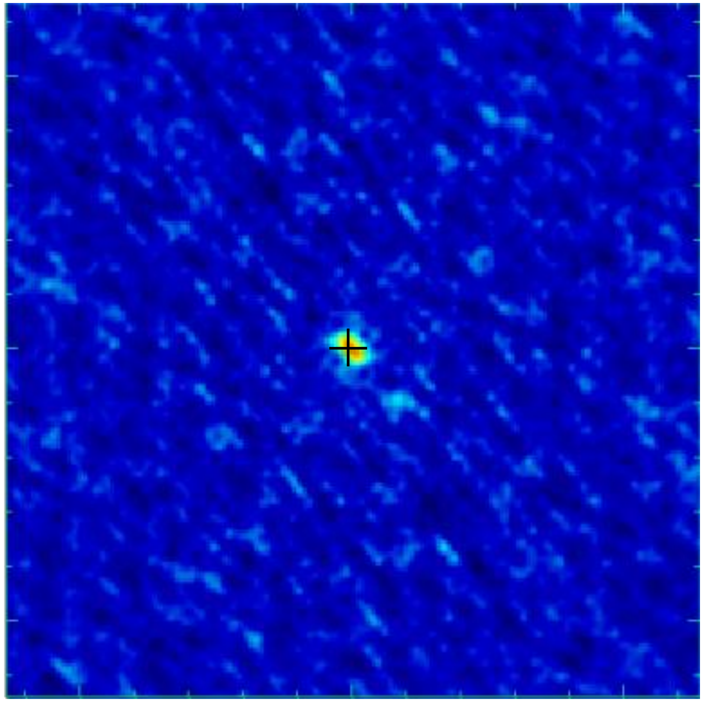


Deconvolution with CLEAN algorithm

Basic algorithm:

Initialize: *residual map* = *dirty map* and list of δ-components = empty

1. Find the peak in the residual map, identify it as a point source
2. Subtract this point source, scaled by *loop_gain* and convolved with the interferometer beam, from the residual image
3. Save the position and subtracted flux to the list of δ-components
4. If stopping criteria are not met, go to step 1



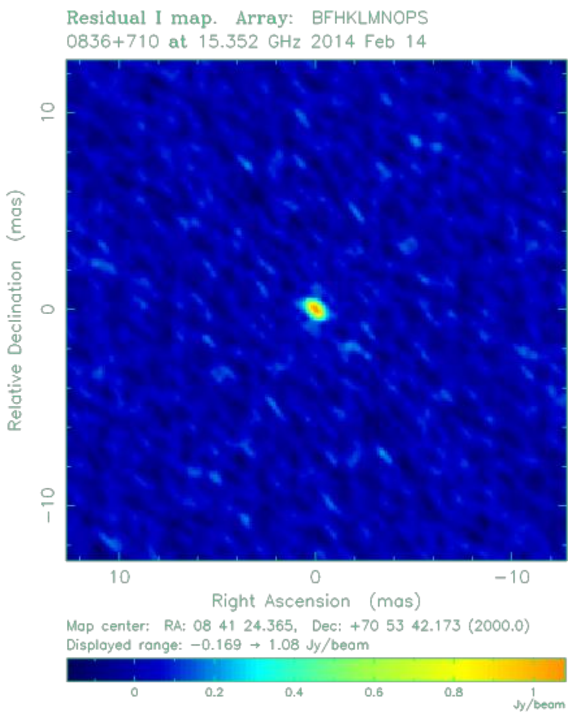
Deconvolution with CLEAN algorithm

- Stopping criteria? Target noise level reached, target SNR reached, or some maximum number of iterations reached.
- Final step – make “restored” image:
  - Make a model image from the final list of δ-components
  - Convolve the model image with a “CLEAN beam”, which is typically a Gaussian fitted to the central peak of the interferometer beam
  - Add the last residual map to present the noise
- The resulting image is an estimate of $I(l, m)$.
- The units are typically Jy / clean_beam_area.

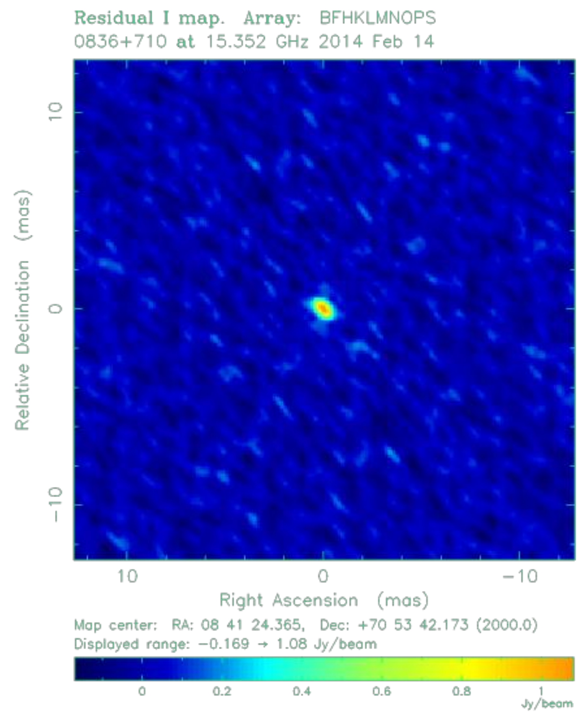
CLEAN example

CLEAN iterations = 0

Dirty image



Residual image

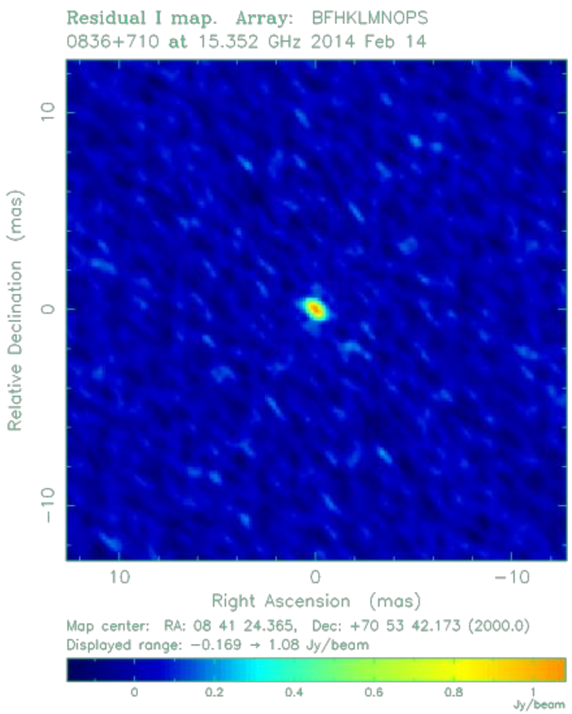


CLEAN image (log)

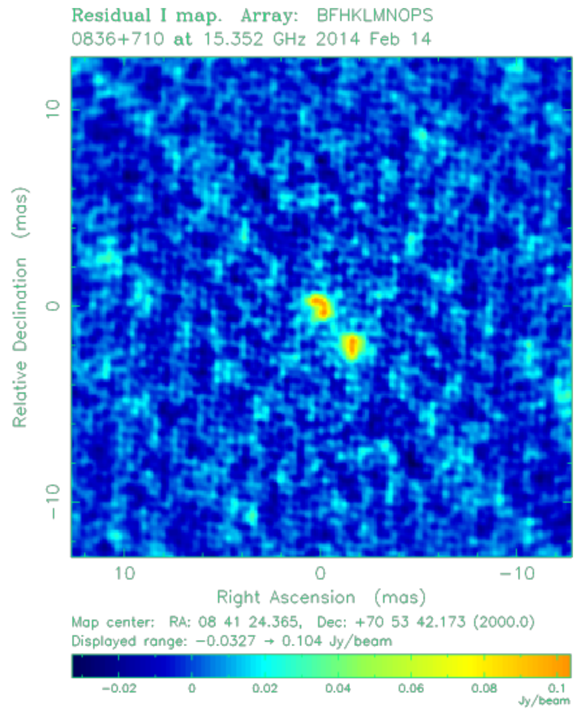
CLEAN example

CLEAN iterations = 100

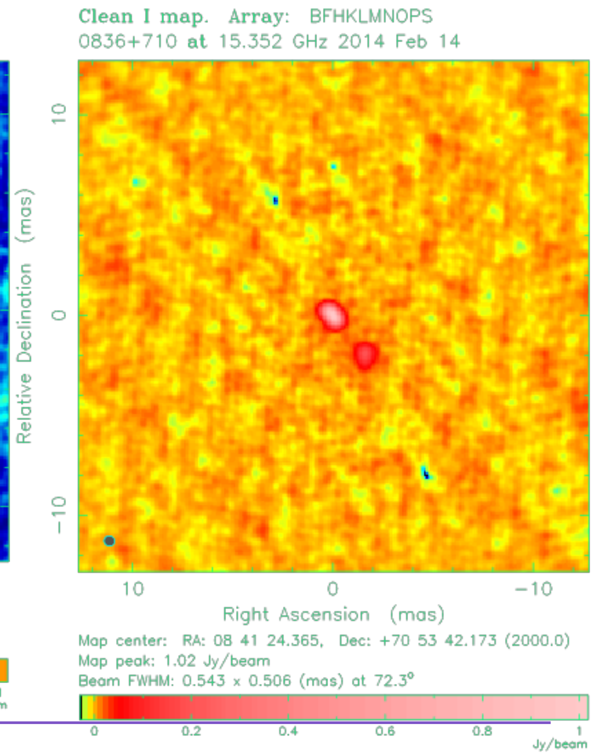
Dirty image



Residual image



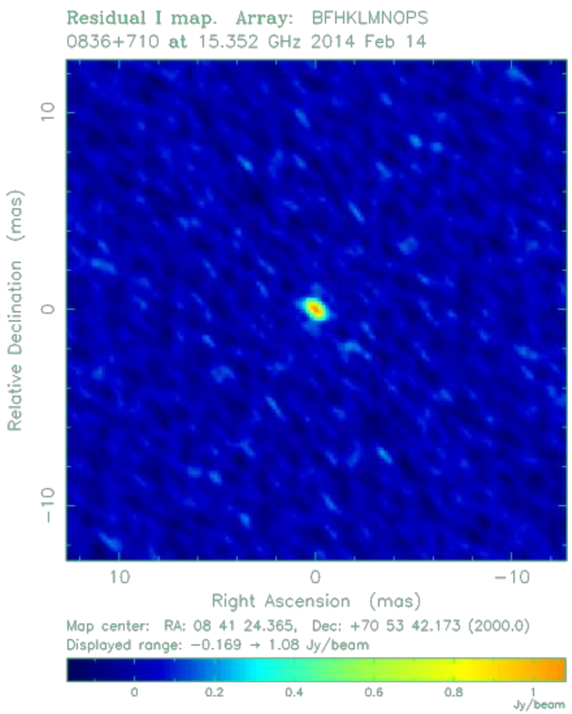
CLEAN image (log)



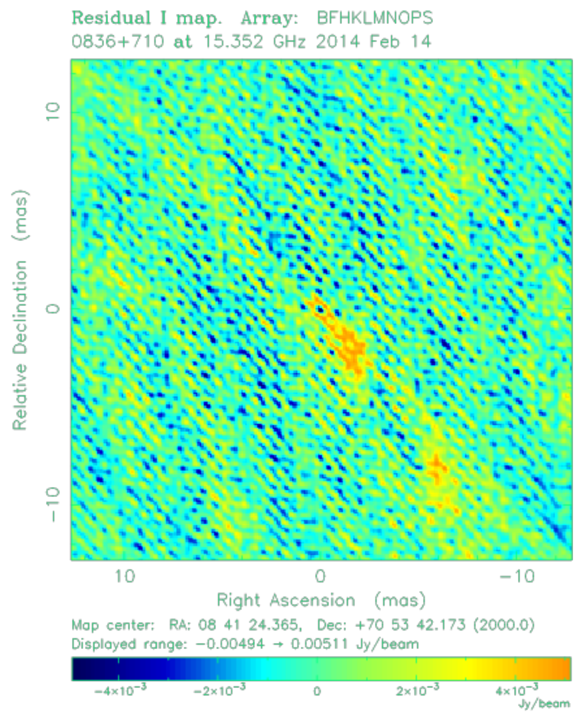
CLEAN example

CLEAN iterations = 500

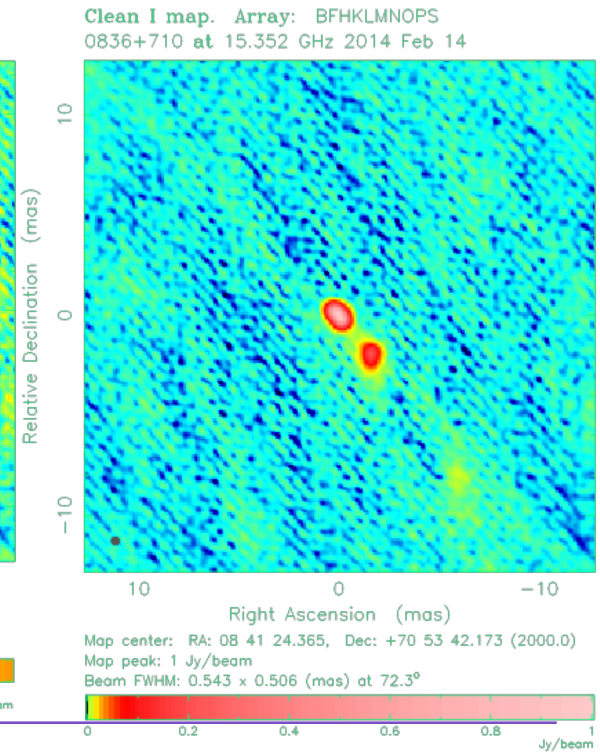
Dirty image



Residual image



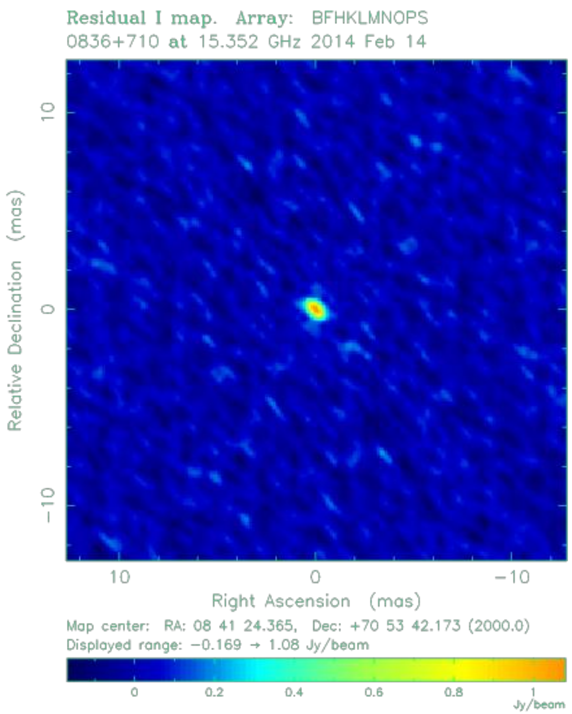
CLEAN image (log)



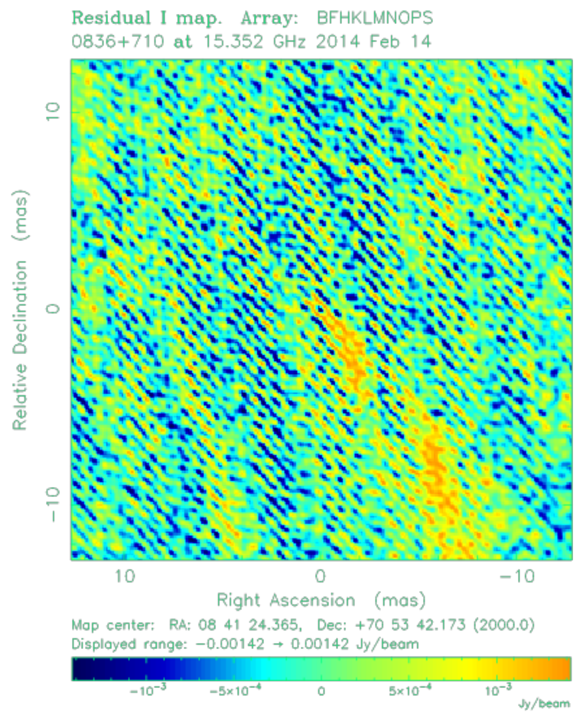
CLEAN example

CLEAN iterations = 1500

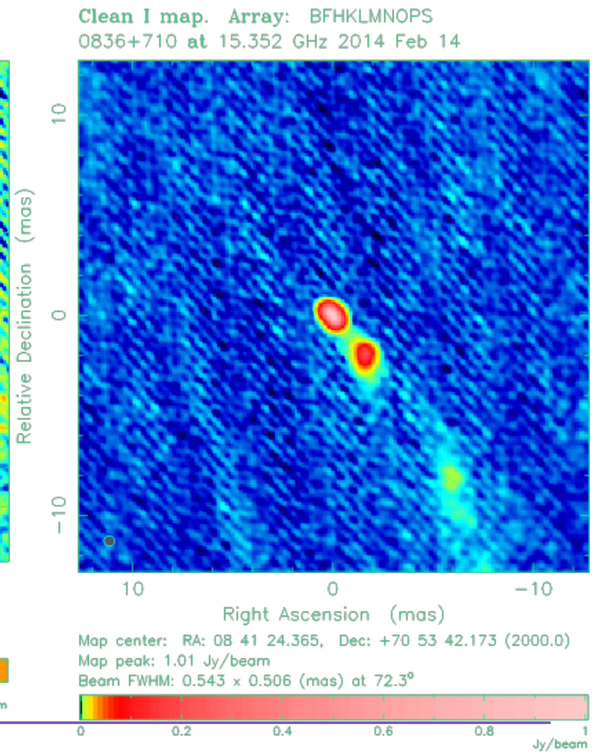
Dirty image



Residual image



CLEAN image (log)



Other deconvolution methods

Maximum entropy method (MEM)

- Assumes that $I(l,m)$ is as smooth as possible
- Minimizes the pixel variance, while keeping χ^2 of the fit acceptable
- Works better than CLEAN in extended and diffuse sources
- Fails to remove sidelobes, if there is a point source on top of an extended source

Multi-scale CLEAN

- Promising results for extended emission

Non-negative least squares

- Directly solves for the (point source) model parameters assuming positivity
- If the source is small, a unique solution may exist

Compressed sensing methods

- Based on sparsity of the data, implementations minimize L_1-norm

Reading (and watching) material

- **Thompson, Moran & Swenson: *"Interferometry and Synthesis in Radio Astronomy"*, Wiley (2004)** https://link.springer.com/book/10.1007/978-3-319-44431-4
- **Taylor, G. B., Carilli, C. L. & Perley, R. A.: "*Synthesis Imaging in Radio Astronomy II"* ASP Conference Series Vol. 180 (1999)**
  - *Contents available online, look in the NASA ADS*
- **J. A. Zensus, P. J. Diamond, and P. J. Napier*: "Very Long Baseline Interferometry and the VLBA"* ASP Conference Series, Vol. 82, (1995)**
  - *Book available online: http://www.cv.nrao.edu/vlbabook/*
- **NRAO Synthesis imaging school 2014 lectures are online**
  - *https://science.nrao.edu/science/meetings/2014/14th-synthesis-imaging-workshop*