PET Reconstruction

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Learning objectives

After the the lecture you should know the basic concepts of:

▪Reconstruction workflow: from raw data to images, data corrections, practical examples

▪Analytical and iterative reconstruction algorithms: FBP, MLEM, OSEM

Definitions

- LOR = Line of response between two detectors

- TOF = Time of flight

- List-mode data, sinogram = PET raw data used for data representation and reconstruction

- FBP = Filtered BackProjection, analytical reconstruction algorithm

- MLEM = Maximum Likelihood Expectation Maximum, iterative reconstruction algorithm

- OSEM = Ordered Subset Expectation Maximum, iterative reconstruction algorithm

Further information

Contact people at Turku PET Centre
- **Researcher, Jarmo Teuho (jarmo.teuho@tyks.fi)**
- **Assistant Prof. Riku Klen (riku.klen@utu.fi)**

OMEGA
- https://github.com/villekf/OMEGA

CASToR
- https://www.castor-project.org/

STIR/SIRF
- http://stir.sourceforge.net/
- https://www.ccppetmr.ac.uk/node/1

NiftyRec
- http://niftyrec.scienceontheweb.net/wordpress/

CASToR
- https://castor-project.org/

ASIM/SIMSET
- http://depts.washington.edu/simset/html/user_guide/user_guide_index.html
- https://depts.washington.edu/asimuw/

GATE/GEANT
- http://www.opengatecollaboration.org/

Homepage of J.Fessler:
- https://web.eecs.umich.edu/~fessler/

Adam Kesners Medical Physics Pages:
- https://sites.google.com/a/fulbrightmail.org/kesnersmedicalphysics/home/education/3d-image-reconstruction-explained-with-animated-gifs

Texbooks for Later Reading

IAEA Medical Physics Handbooks
- **Radiation Oncology Physics:**
  https://www.iaea.org/publications/7086/radiation-oncology-physics
- **Diagnostic Radiology Physics:**
  https://www.iaea.org/publications/8841/diagnostic-radiology-physics
- **Nuclear Medicine Physics:**
  - **https://www.iaea.org/publications/10368/nuclear-medicine-physics**

Radiation Oncology Physics · Diagnostic Radiology Physics · Nuclear Medicine Physics
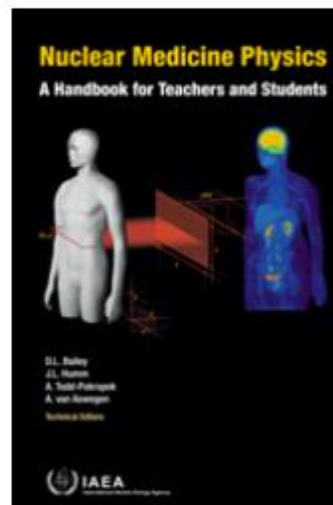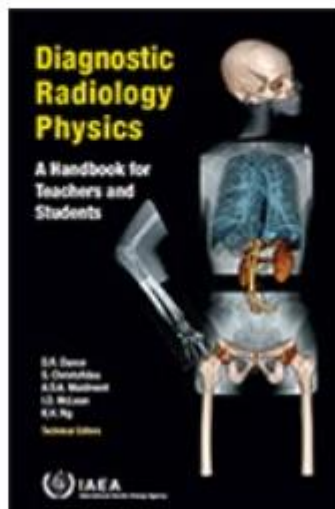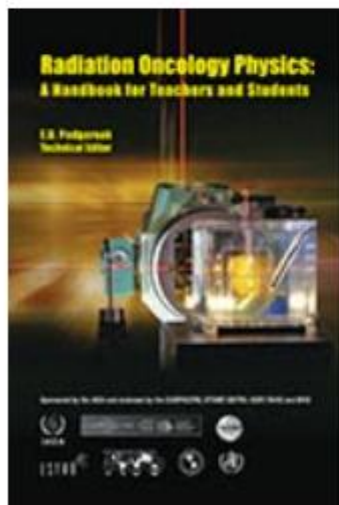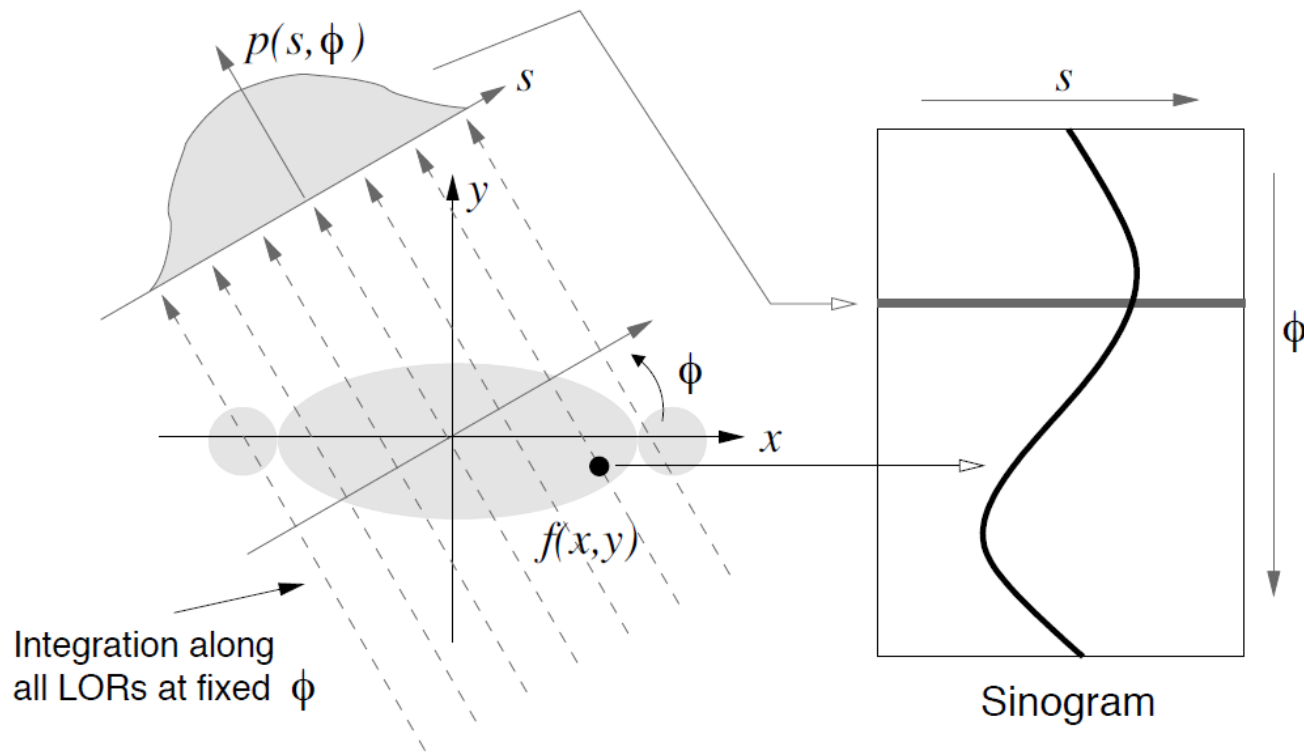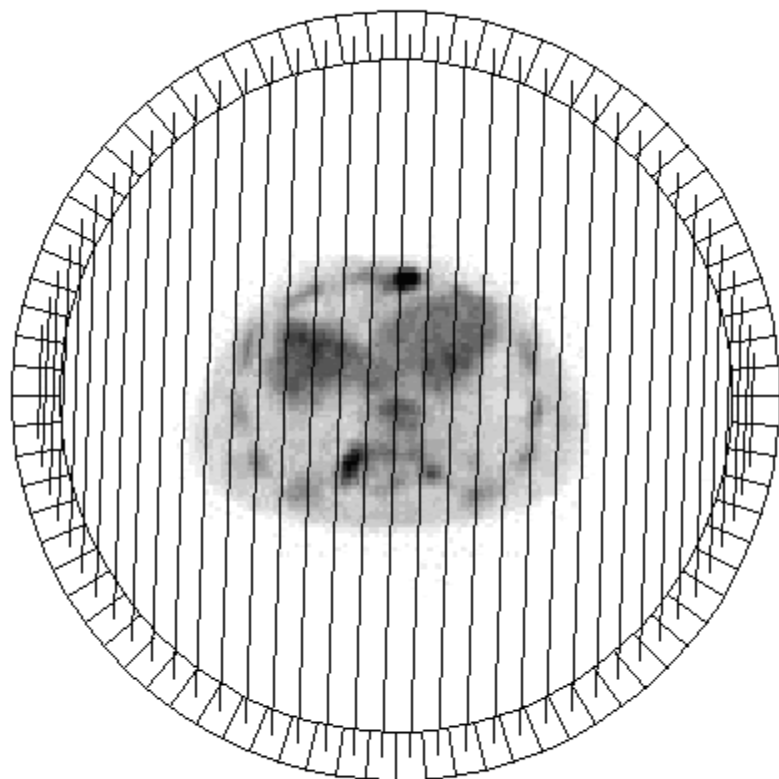
Sinogram Formation, Forward Projection

Figure 2. A projection, $p(s,\phi)$, is formed from integration along all parallel LORs at an angle ϕ. The projections are organized into a sinogram such that each complete projection fills a single row of ϕ in the sinogram. In this format, a single point in $f(x,y)$ traces a sinusoid in the sinogram.

PET Image Reconstruction
Adam Alessio and Paul Kinahan

Lines of response between PET detectors

Corresponding location in sinogram

Angle: 0 °

Theta (angle)

Rho (offset)

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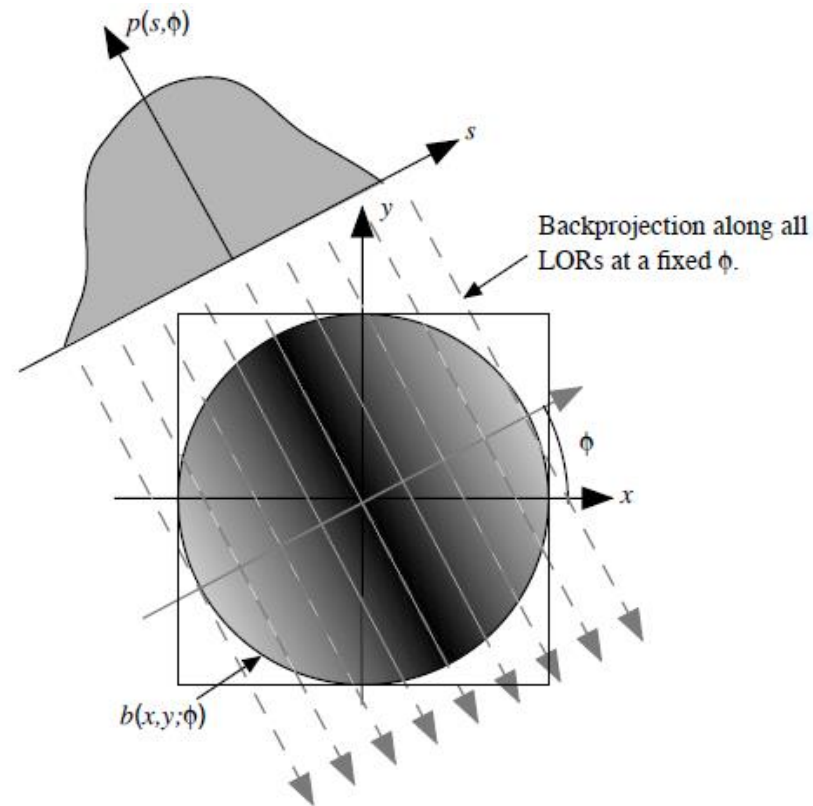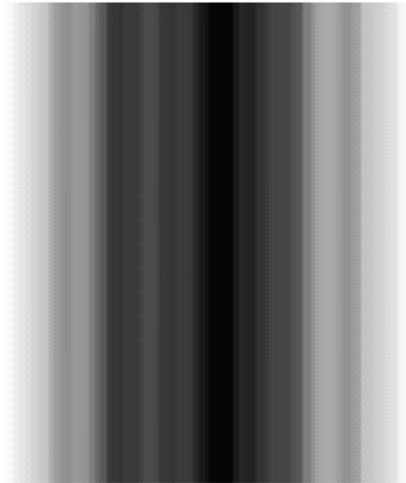
Backprojection

Figure 6. Backprojection, $b(x, y; \phi)$, into an image reconstruction array of all values of $p(s, \phi)$ for a fixed value of ϕ.

PET Image Reconstruction
Adam Alessio and Paul Kinahan

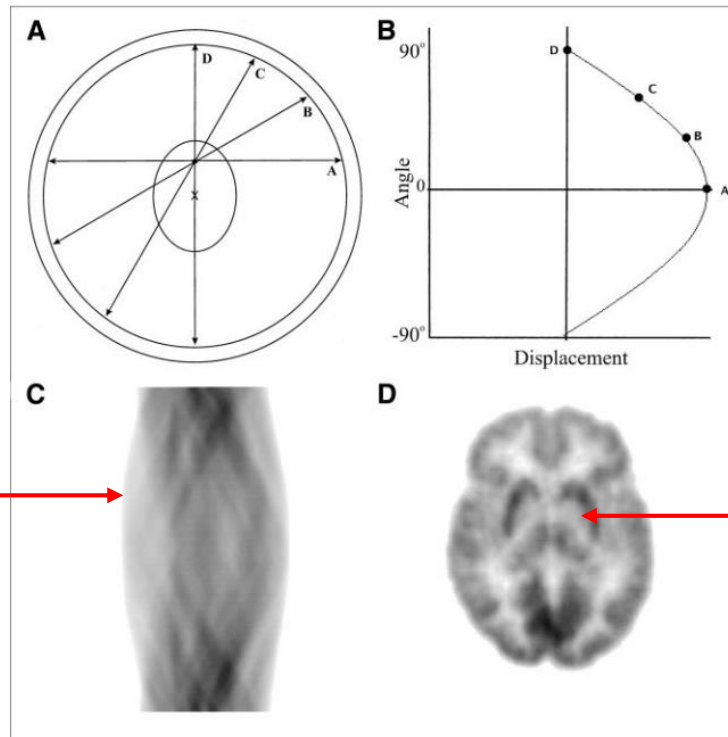Reconstructed image · Sinogram · Theta (angle) · Rho (offset) · Back Projection

Practical Example One 2D Sinogram

FIGURE 1. Sinogram formation. Coincidence events in PET scanner are categorized by plotting each LOR as function of its angular orientation versus its displacement from center of gantry. (A) Center of gantry is noted by cross (X), and locus of interest (e.g., tumor) is noted by ellipse. Four LORs passing through locus of interest are labeled A, B, C, and D. (B) These 4 LORs are plotted on this sinogram where angular orientation is on y-axis and displacement from center of gantry is on x-axis. If all possible LORs that pass through this point are plotted, it maps out half of sine wave turned on its side as shown here. (C) Sinograms of more complicated objects, such as sinogram of brain scan shown, are composed of many overlapping sine waves. (D) Reconstructed brain image corresponding to sinogram in (C) is shown.

The sinogram is a two-dimensional histogram of the LORs in distance and angle coordinates

Each sinogram represents the data acquired for a slice across all angles

After data corrections and image reconstruction, the image is a pixel-by-pixel representation of the radiotracer concentration at scan time.

Data Acquisition in PET Imaging, Frederic H. Fahey

Reconstruction Workflow

❑ Data corrections

❑ Workflow from raw data to PET images

❑ Practical example with LM-OSEM

Data Corrections

❑ Power of PET is in quantification, which requires accurate **data corrections** to be implemented

❑ With analytical algorithms such as FBP the data are corrected before reconstruction

❑ Corrections are incorporated in the reconstruction loop in iterative algorithms, such as OSEM:
  ▪ Detector geometry, normalisation of detector efficiencies, detector dead-time
  ▪ Attenuation, scatter, random events
  ▪ Resolution recovery (PSF), TOF-specific corrections
  ▪ Image regularisation parameters

❑ Calibration from counts to activity units (kBq/ml) and correcting for radioactive decay are also performed

❑ Post-smoothing (e.g. 3D Gaussian with FHWM in mm) is applied to control the noise in the images, although some expections apply (e.g. Q.Clear, blob-RAMLA)
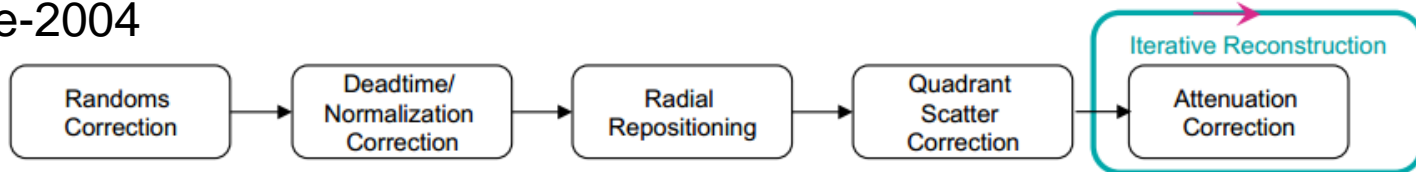
From Raw Data to PET Images

Sort to sinograms

Data corrections
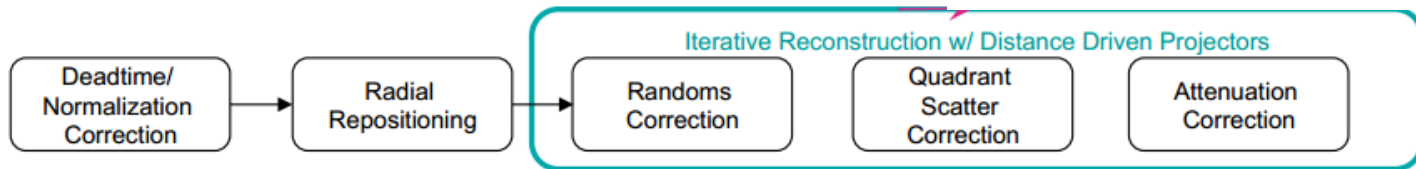
Final corrected sinogram

Final, quantitative PET image
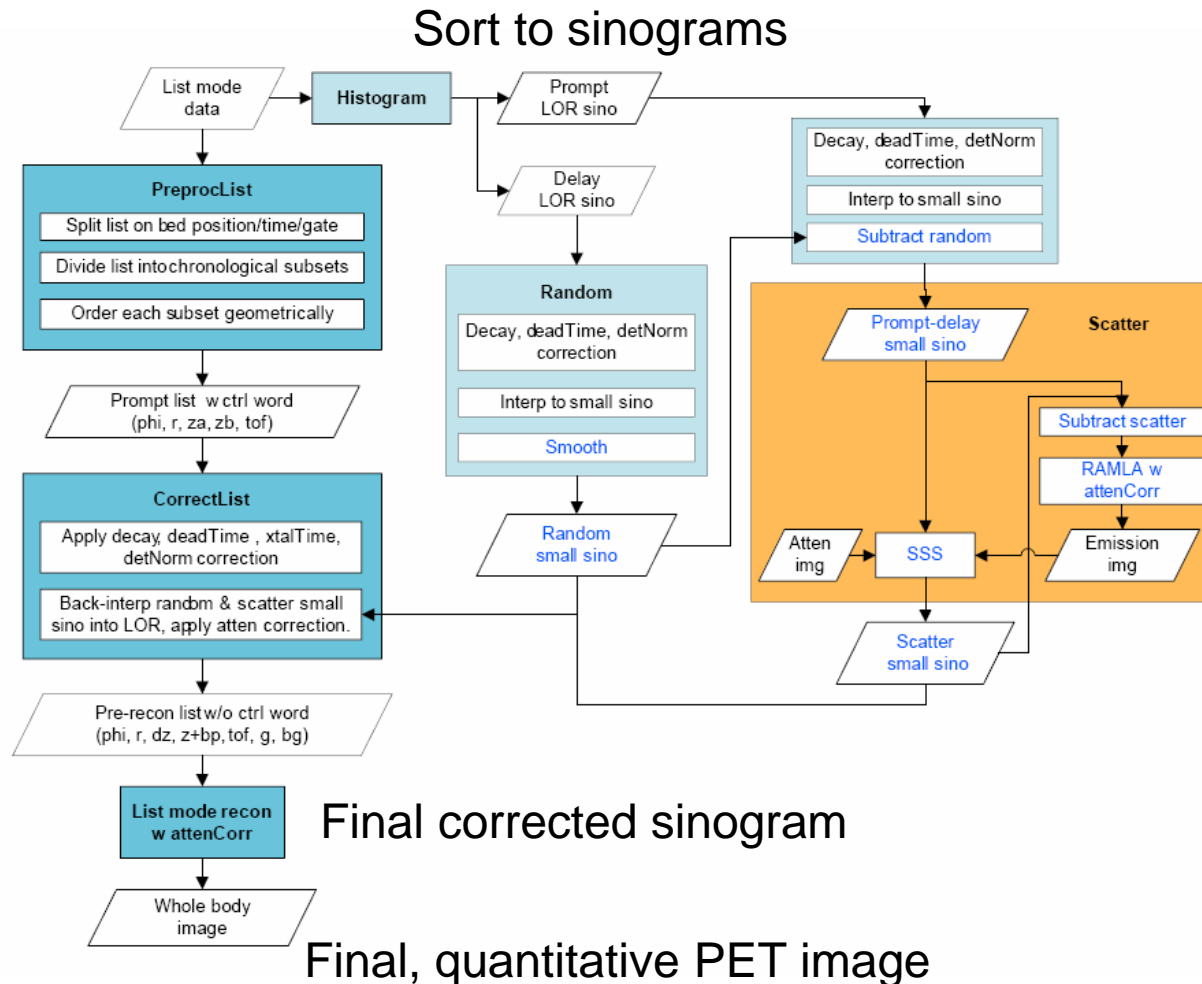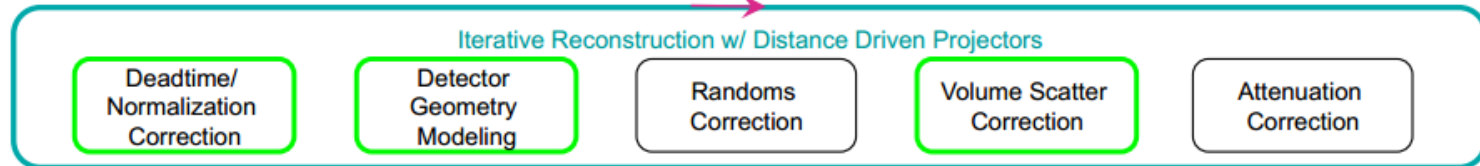
Data Corrections in Image Reconstruction

Pre-2004

$$f_i^{n+1} = f_i^n \frac{1}{\sum_{j'=1}^{N_{LOR}} a_{j',i} / \alpha_{j'}} \sum_{j=1}^{N_{LOR}} a_{j,i} \frac{p_j}{\sum_{i'=1}^{P} a_{j,i'} f_{i'}^n}$$

$$i = 1, \ldots, P$$

$$f_i^{n+1} = f_i^n \frac{1}{\sum_{j'=1}^{N_{LOR}} a_{j',i} / \alpha_{j'}} \sum_{j=1}^{N_{LOR}} a_{j,i} \frac{p_j + 2\bar{r}_j + \bar{s}_j}{\sum_{i'=1}^{P} a_{j,i'} f_{i'}^n + \alpha_j (2\bar{r}_j + \bar{s}_j)}$$

$$i = 1, \ldots, P$$

Mid-2007

Practical Example LM-OSEM

A relaxed List Mode Ordered Subset Expectation Maximization (LMOSEM) algorithm

$$\hat{f}_i^{k,m} = \hat{f}_i^{k,m-1} \left\{ (1-\lambda) + \frac{\lambda}{s_i} \sum_{j \in subset\ m} \eta_j^{atten} H_{ji}^{TOF} \frac{1/\eta_j^{multi}}{\eta_j^{atten} \sum_{n=0}^{N-1} H_{jn}^{TOF} \hat{f}_n^k + b_j^{add}} \right\}$$

Algorithm-specific lambda

Attenuation correction

System matrix, TOF-specific

$$where \quad s_i = \sum_{all\ possible\ j \in subset\ m} \eta_j^{atten} H_{ji}^{TOF}$$

$$\eta_j^{multi} = \eta_j^{xtaleff} \eta_j^{det\ geom} \eta_j^{decay} \eta_j^{deadtime} \eta_j^{xtaltime}$$

$$b_j^{add} = \left(sc_j(\hat{f}^k, \mu) + r_j \right) / \eta_j^{det\ geom}$$

Sensitivity image/system matrix, TOF-spe

Multiplicative correction factors

Additive corrections: scatter, randoms

Where Do I Get a System Model?

Contents lists available at ScienceDirect

Computerized Medical Imaging and Graphics

journal homepage: www.elsevier.com/locate/compmedimag

Review

System models for PET statistical iterative reconstruction: A review

A. Iriarte[a,*], R. Marabini[b], S. Matej[c], C.O.S. Sorzano[a,d], R.M. Lewitt[c]

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[b] Escuela Politécnica Superior, Universidad Autónoma de Madrid, 28049 Madrid, Spain
[c] Department of Radiology, University of Pennsylvania, Philadelphia, PA 19104, USA
[d] National Center of Biotechnology (CSIC), 28049 Madrid, Spain

Reconstruction Algorithms

❑ Analytical vs Iterative

❑ FBP

❑ MLEM

❑ OSEM

Analytical Reconstruction Algorithms

❑ One way to represent the imaging system is with the following linear relationship:
- p = Hf +n, where p is the set of projections, H is the known system model, f is the unknown image, and n is the error in the observations.

❑ Analytical reconstruction techniques use the inverse of the discrete Radon transform to solve this problem, offering a <u>direct mathematical solution</u> for the image f from measured projection p.
- Advantages: **<u>scales linearly</u>** with the acquired counts
- Disadvantages: system model H is assumed ideal, poor noise handling/propagation, streak artifacts

❑ **Background reading** (e.g. Two-dimensional central slice theorem):
- Image Reconstruction Algorithms in PET, Michel Defrise, Paul E Kinahan and Christian J Michel
- PET Image Reconstruction, Adam Alessio and Paul Kinahan

Filtered Backprojection (FBP)

❑ Most frequently used analytic reconstruction algorithm is the filtered backprojection (FBP) algorithm

❑ The goal of FBP reconstruction is to compute the the image f(x,y) from projections p(s,φ) by:

$$f(x,y) \approx \tilde{f}(x,y) = \int\limits_{0}^{\pi} \mathcal{F}_1^{-1}\left\{W(v_s)|v_s|\mathcal{F}_1\{p(s,\phi)\}\right\}d\phi$$

❑ In algorithmic form:
- 1D Fourier transform of p(s,φ)
- Multiplication with a ramp filter v_s
- Apply a smoothing function/window $W(v_s)$
- Inverse Fourier transform of the filtered projections
- Backproject across all angles to image space

❑ We will consider a 2D case in the next slides, specific considerations apply to 3D case => **homework** ☺

FBP, MATLAB Example Noiseless data

1. for all angles φ:
   a) 1D Fourier transform into frequency space
      $$p(x_r, \varphi) \rightarrow P(v_{x_r}, \varphi)$$
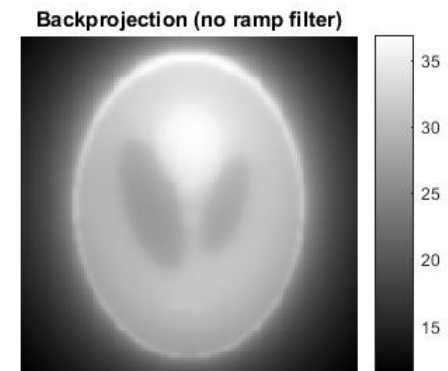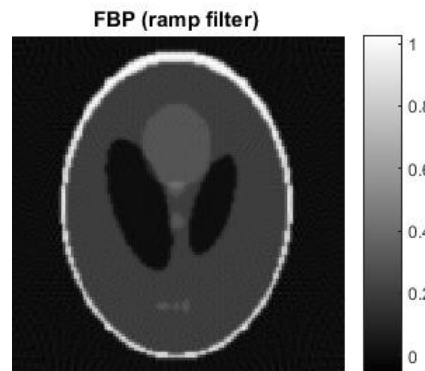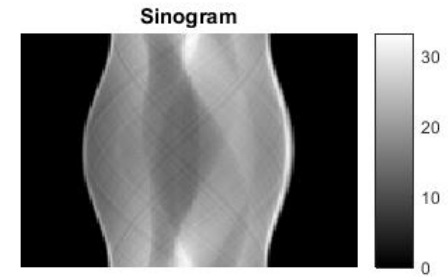   b) multiplication with ramp filter v_{x_r}:
      $$P^F(v_{x_r}, \varphi) = P(v_{x_r}, \varphi)\, v_{x_r}$$
   c) inverse 1D Fourier transform of filtered projections into projection space:
      $$P^F(v_{x_r}, \varphi) \rightarrow p^F(x_r, \varphi)$$

2. backprojection $f(x, y) = \int_0^P p^F(x_r, \varphi) d\varphi$

Performing mere backprojection versus applyinging backprojection after multiplication with a ramp filter in Fourier space.

Reference object

Sinogram

FBP (ramp filter)

Backprojection (no ramp filter)

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FBP, MATLAB Example Noisy Data

1. for all angles φ:
   a) 1D Fourier transform into frequency space
      $$p(x_r, \varphi) \rightarrow P(v_{x_r}, \varphi)$$
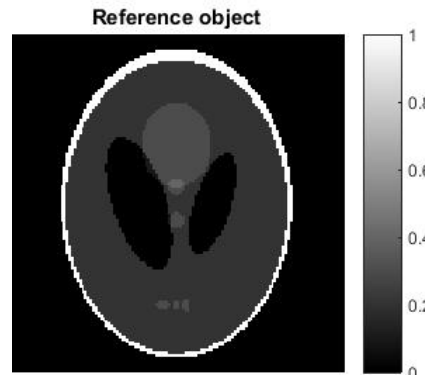   b) multiplication with ramp filter v_{x_r}:
      $$P^F(v_{x_r}, \varphi) = P(v_{x_r}, \varphi)\, v_{x_r}$$
   c) inverse 1D Fourier transform of filtered projection into projection space:
      $$P^F(v_{x_r}, \varphi) \rightarrow p^F(x_r, \varphi)$$

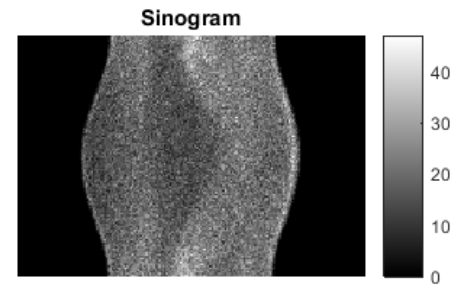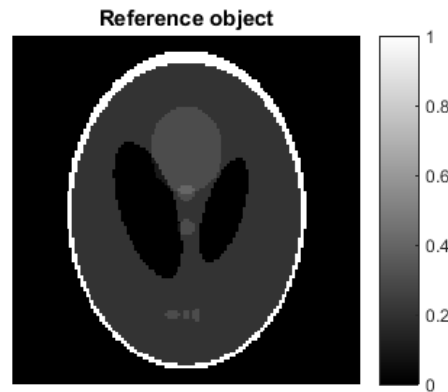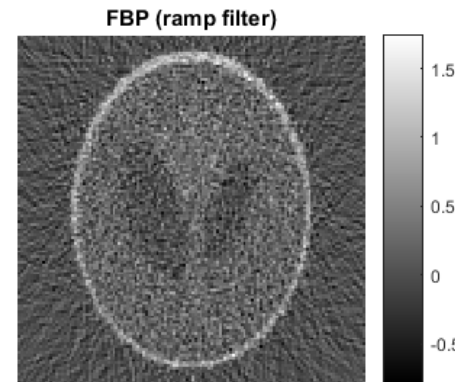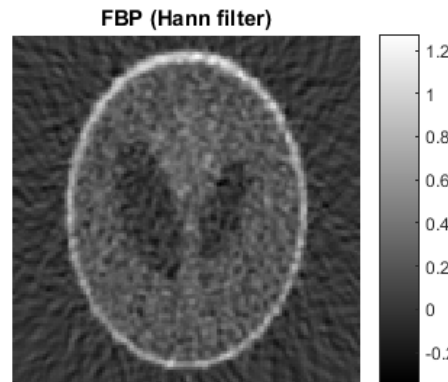2. backprojection $f(x, y) = \int\limits_0^P p^F(x_r, \varphi)\,d\varphi$

Applying additional filtering windows: Hann, Hamming, Shepp-Logan, etc.

Noise regularization is frequency selective (cutoff & window) => trade-off between resolution and noise.

Reference object

Sinogram

FBP (Hann filter)

FBP (ramp filter)
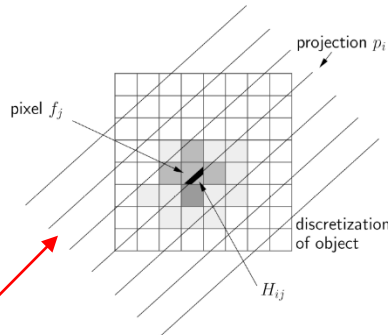
Iterative Reconstruction Techniques

- ❑ Iterative reconstruction, benefits:
    - ▪ Can model the noise in the measurements + include a realistic system model
    - ▪ Improved image quality (contrast, lesion detection, ..)
    - ▪ Reduced / no streak artifacts, potentially less sensitive to missing data

- ❑ Iterative reconstruction, pitfalls:
    - ▪ Greater computational demands
    - ▪ **Non-linear behavior** => FBP is considered "gold standard" in certain measurements

- ❑ Basic components:
    - ▪ Model for the image (pixels, voxels, blobs)
    - ▪ Model for the system H (characterizing the imaging system) that relates image to data (probability that an emission from voxel is detected in projection)
    - ▪ Model for the data (statistical relationship between the measurements and expected value, e.g. a Poisson model) => objective function
    - ▪ Governing principle that defines the "best" image (e.g. Maximum Likelihood = ML)
    - ▪ Final component: **algorithm** that finds the best image estimate (e.g. an image, which is a solution of a maximization of an objective function => maximizes the Poisson-log likelihood function)
    - ▪ **In general, the algorithms themselves are only discussed/referred**
        - • **Two widely used approaches for finding the ML estimate: MLEM and OSEM**
        - • **Apply to both 2D and 3D data (e.g. 2D-OSEM, 3D-OSEM)**

MLEM

Image and system model

$$\bar{p}_i = \sum_{j=1}^{N} H_{ij} f_j$$

projection p_i

pixel f_j

discretization of object

H_{ij}

Figure 9. Illustration of a single element of the system model H_{ii}.

Detection process is modeled in the system matrix

Poisson Likelihood function, model for the data

$$\mathbf{L}(P = p \mid f) = \prod_{i=1}^{M} \frac{\bar{p}_i^{\,p_i} \exp(-\bar{p}_i)}{p_i!} \, .$$

Search for an image f that makes the measured data most likely to occur at argmax() of Poisson log-likelihood

Algorithm

Image at n^{th} iteration

Measured data (emission sinogram)

$$f_j^{(n+1)} = \frac{\hat{f}_j^{(n)}}{\sum_{i'} H_{i'j}} \sum_{i} H_{ij} \frac{p_i}{\sum_{k} H_{ik} \hat{f}_k^{(n)}}$$

Updated image

System matrix

Forward projected image at n^{th} iteration

MLEM
Example of One Iteration

Initial estimate

Updated image

Updated image

Forward projected estimate

Forward projected estimate

Backprojected estimate

Image Domain

repeat until reach final estimate

$$\hat{f}^{(0)} \rightarrow \hat{f} \rightarrow$$

4) Update Image (and Weight Based on System Matrix)

Projection Domain

1) Forward Project to all projections

$$\sum_k H_{ik} \hat{f}_k^{(n)}$$

2) Compare with measured projections

$$\frac{p_i}{\sum_k H_{ik} \hat{f}_k^{(n)}}$$

3) Backproject ratio to all voxels
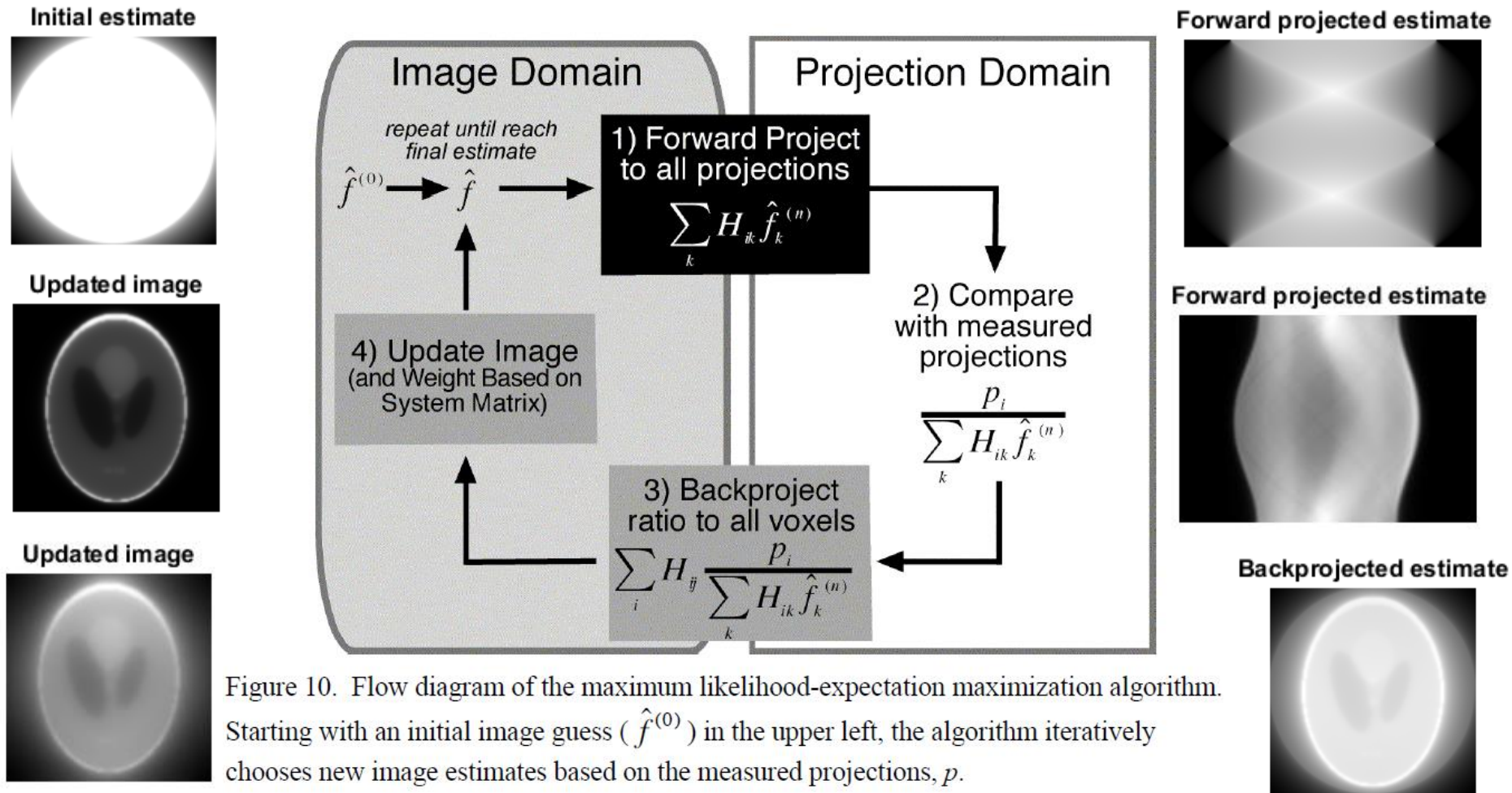
$$\sum_i H_{ij} \frac{p_i}{\sum_k H_{ik} \hat{f}_k^{(n)}}$$

Figure 10. Flow diagram of the maximum likelihood-expectation maximization algorithm. Starting with an initial image guess ($\hat{f}^{(0)}$) in the upper left, the algorithm iteratively chooses new image estimates based on the measured projections, p.

OSEM

❑ Ordered Subsets Expectation Maximization (OSEM) was introduced to reduce reconstruction time of conventional MLEM.

❑ OSEM uses subsets of the entire dataset for each image update in the form:

$$f_j^{(n+1)} = \frac{\hat{f}_j^{(n)}}{\sum_{i' \in S_b} H_{i'j}} \sum_{i \in S_b} H_{ij} \frac{p_i}{\sum_k H_{ik} \hat{f}_k^{(n)}}$$

❑ The backprojection steps sum over only the projections in subset **Sb** of a total of **b** subsets, which are non-overlapping.

❑ Therefore, the image is updated during each subiteration and one complete iteration will have **b** image updates, allowing faster convergence over MLEM.

❑ When there is only one subset (**b = 1**), OSEM is the same as MLEM. However, although OSEM resembles MLEM:
  ▪ It is not guaranteed to converge to ML solution (in practice, convergence is similar to MLEM)
  ▪ It has more image variance at the same bias level compared to MLEM

OSEM vs MLEM – FIGHT! MATLAB Example
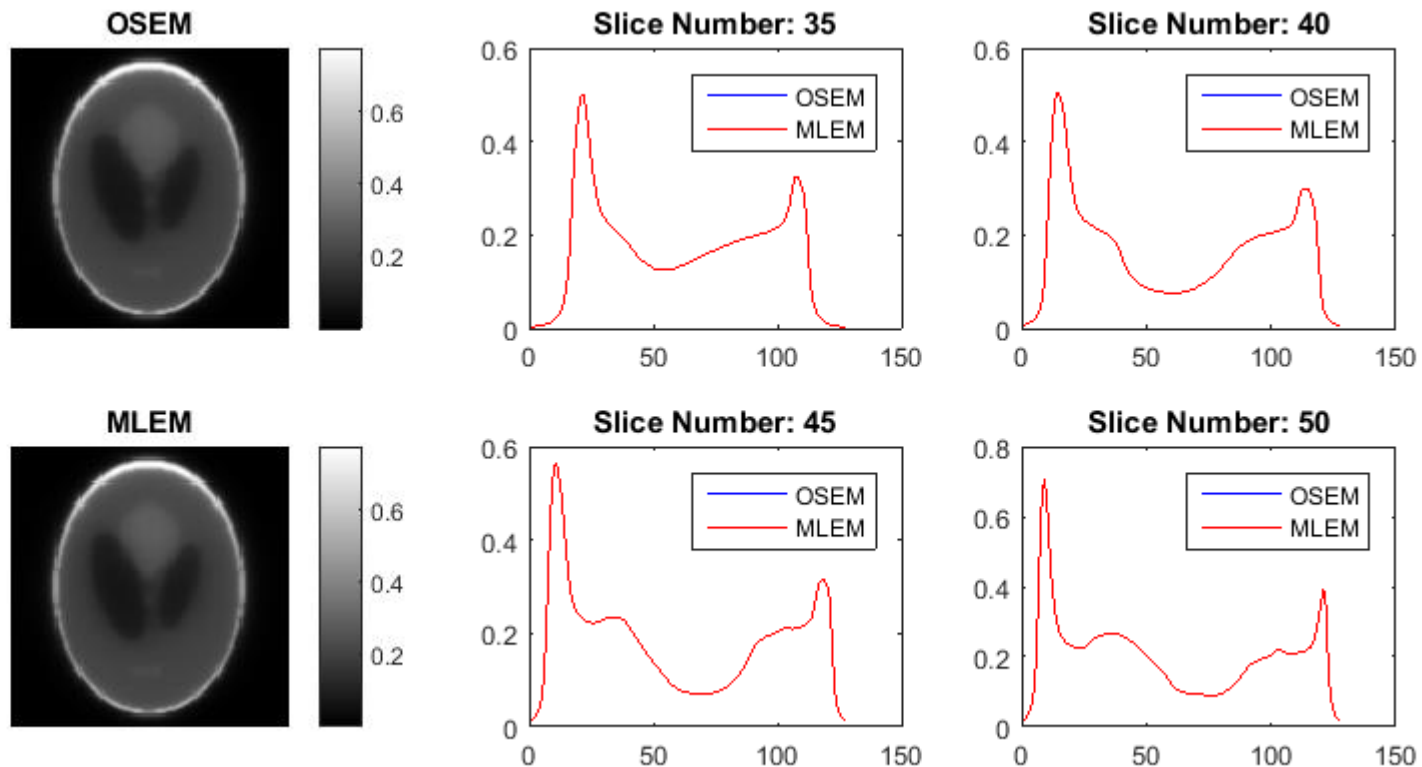
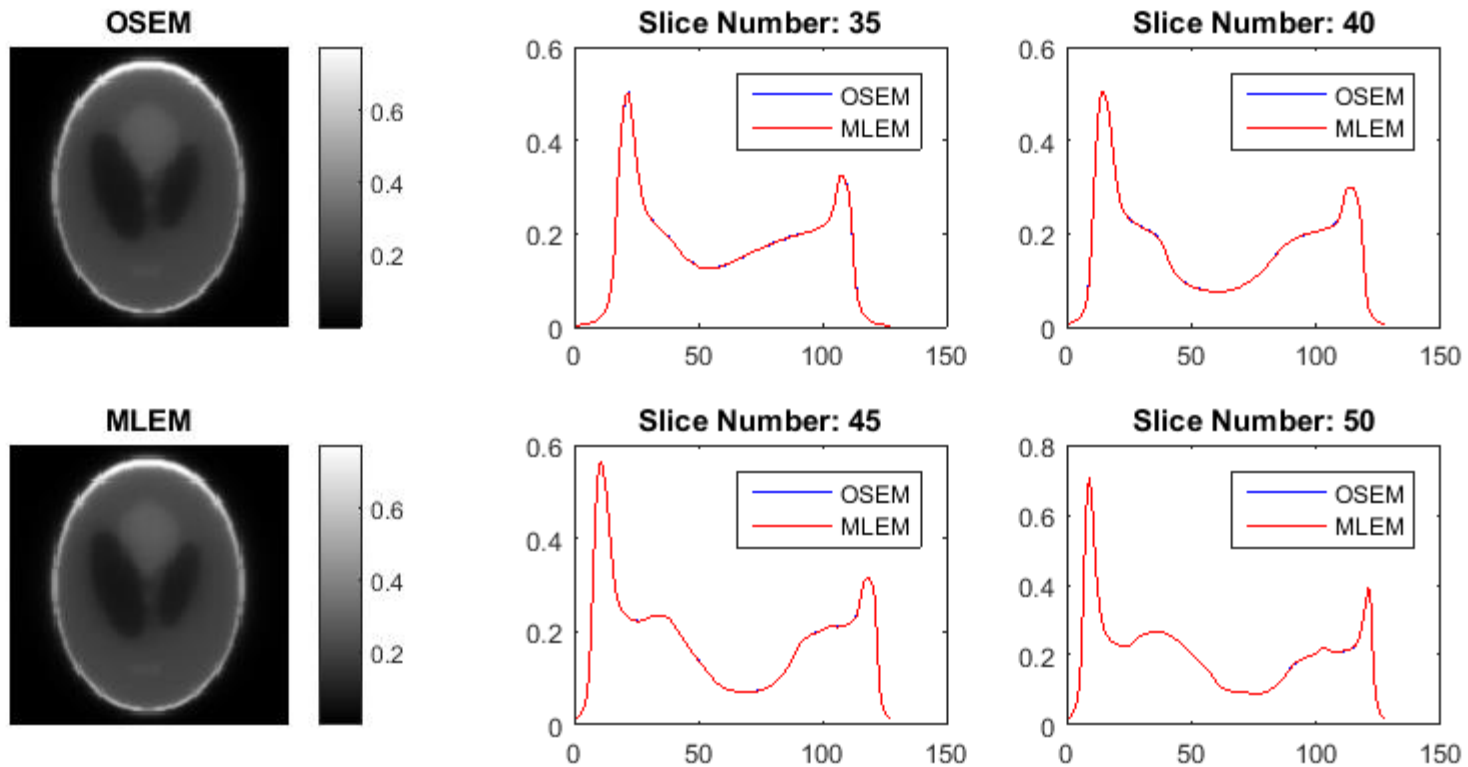OSEM: 1 subset, 8 iterations
MLEM: 8 iterations

OSEM vs MLEM – FIGHT! MATLAB Example

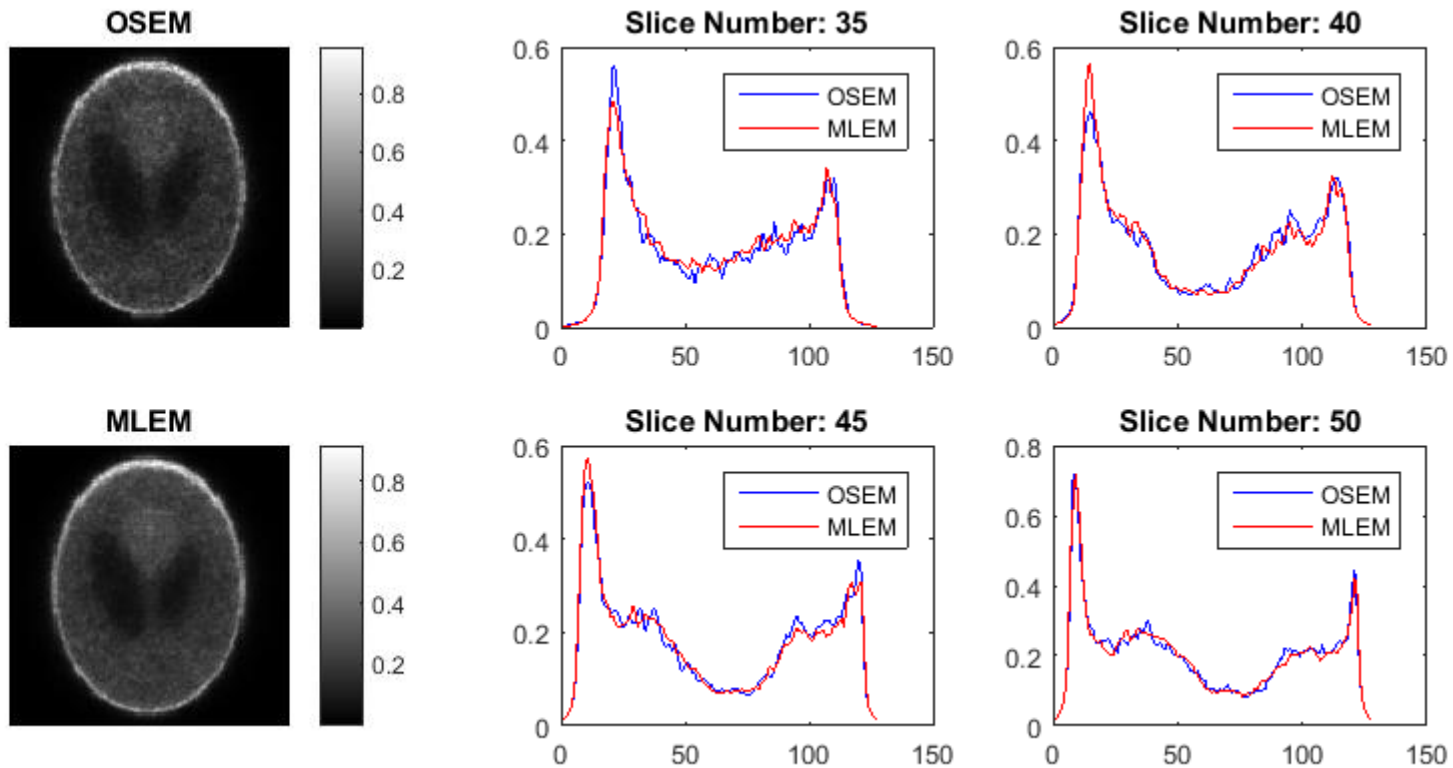OSEM: 2 subsets, 4 iterations
MLEM: 8 iterations

OSEM vs MLEM – FIGHT! MATLAB Example

OSEM: 2 subsets, 4 iterations
MLEM: 8 iterations

Regularised Recon

Accurate and Consistent Lesion Quantitation with Clinically Acceptable Penalized Likelihood Images

Evren Asma[1], *Member, IEEE*, Sangtae Ahn[1], *Member, IEEE*, Steven G. Ross[2],
Anthony Chen[2] and Ravindra M. Manjeshwar[1], *Member, IEEE*
[1]Functional Imaging Laboratory, General Electric Global Research Center, Niskayuna, NY, 12309
[2]General Electric Healthcare, Milwaukee, WI, 53188

$$R(\mathbf{x}) = \sum_{j=1}^{n_v} \sum_{k \in \mathcal{N}_j} w_j w_k \frac{(x_j - x_k)^2}{x_j + x_k + \gamma |x_j - x_k|}$$

$$\Phi(\mathbf{x}) = \sum_{i=1}^{n_d} y_i \log([\mathbf{P}\mathbf{x}]_i + b_i) - ([\mathbf{P}\mathbf{x}]_i + b_i) - \beta \mathbf{R}(\mathbf{x})$$
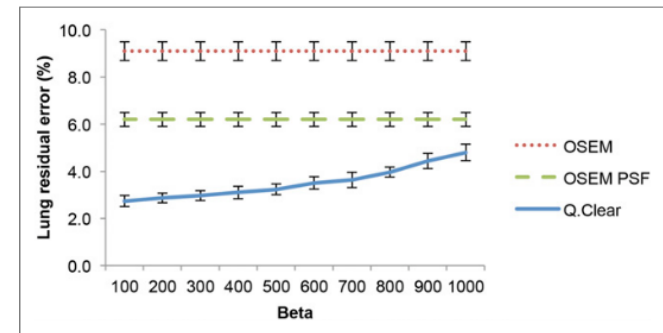
Advantage: full convergion of the reconstructed image

Control the end-appearance of image

Phantom and Clinical Evaluation of the Bayesian Penalized Likelihood Reconstruction Algorithm Q.Clear on an LYSO PET/CT System

Eugene J. Teoh*[1,2], Daniel R. McGowan*[2,3], Ruth E. Macpherson[1], Kevin M. Bradley[1], and Fergus V. Gleeson[1,2]

FIGURE 2. Graph of mean residual LE for OSEM (2 iterations, 24 subsets, 6.4-mm filter), OSEM PSF (3 iterations, 24 subsets, 2-mm filter), and Q.Clear (β = 100–1,000). Error bars shown are 1 SD.
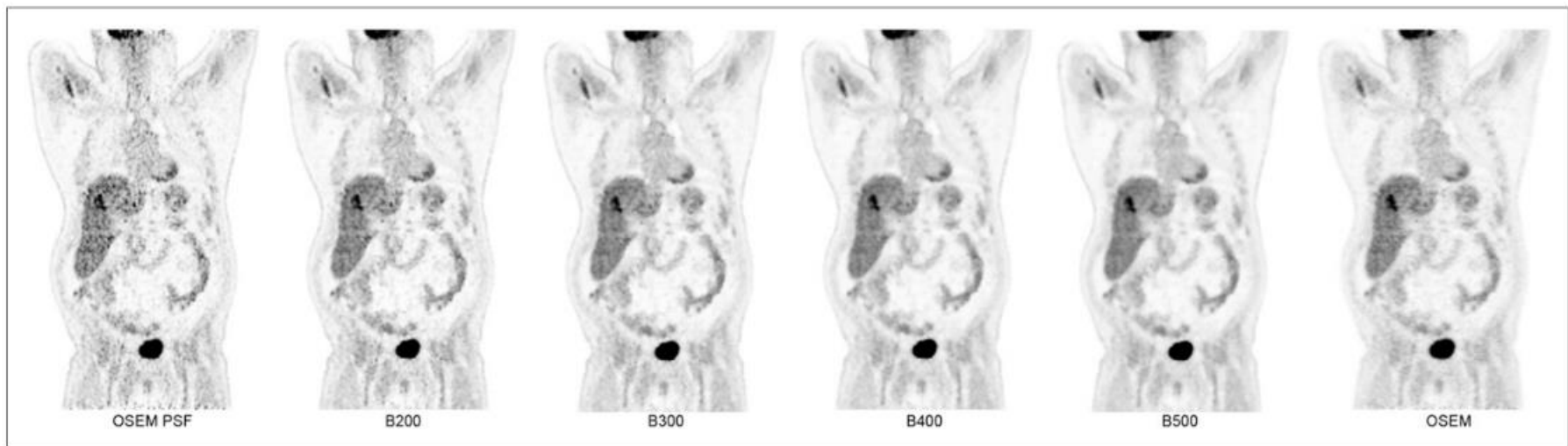
FIGURE 4. Coronal PET images demonstrating ^{18}F-FDG–avid liver metastasis across OSEM and Q.Clear reconstructions. Smooth homogeneous appearance of background liver on Q.Clear reconstruction renders metastasis more conspicuous than OSEM PSF. Conversely, there is also risk of false-positive findings for small foci on OSEM PSF reconstruction because of high level of background noise. All images are displayed on SUV scale 0–6.
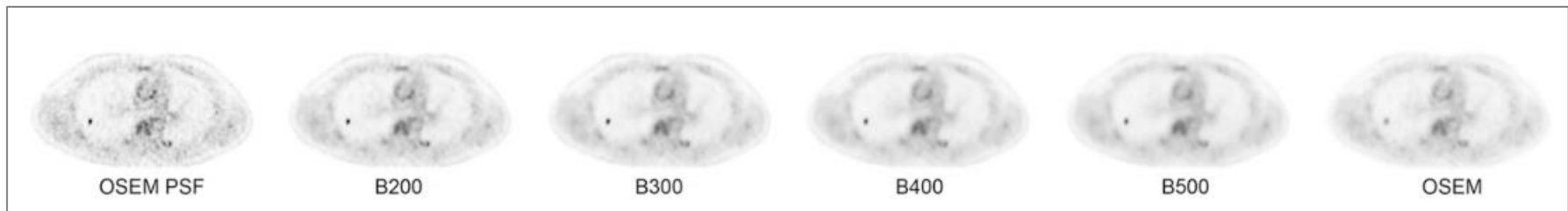

FIGURE 5. Axial PET images demonstrating ^{18}F-FDG–avid 7-mm right upper lobe lung nodule. This was mildly ^{18}F-FDG–avid on OSEM, but degree of uptake was shown to be higher using Q.Clear reconstruction. Despite similar degree of uptake on OSEM PSF, there was still low level of background noise from within lungs, which was reduced even on B200. Improvement in background noise within mediastinum, compared with OSEM PSF, is also illustrated. All images are displayed on SUV scale 0–6.

Teoh, E. J., et al. Phantom and Clinical Evaluation of the Bayesian Penalized Likelihood Reconstruction Algorithm Q.Clear on an LYSO PET/CT System. *J. Nucl. Med.* **56**, 1447–1452 (2015).
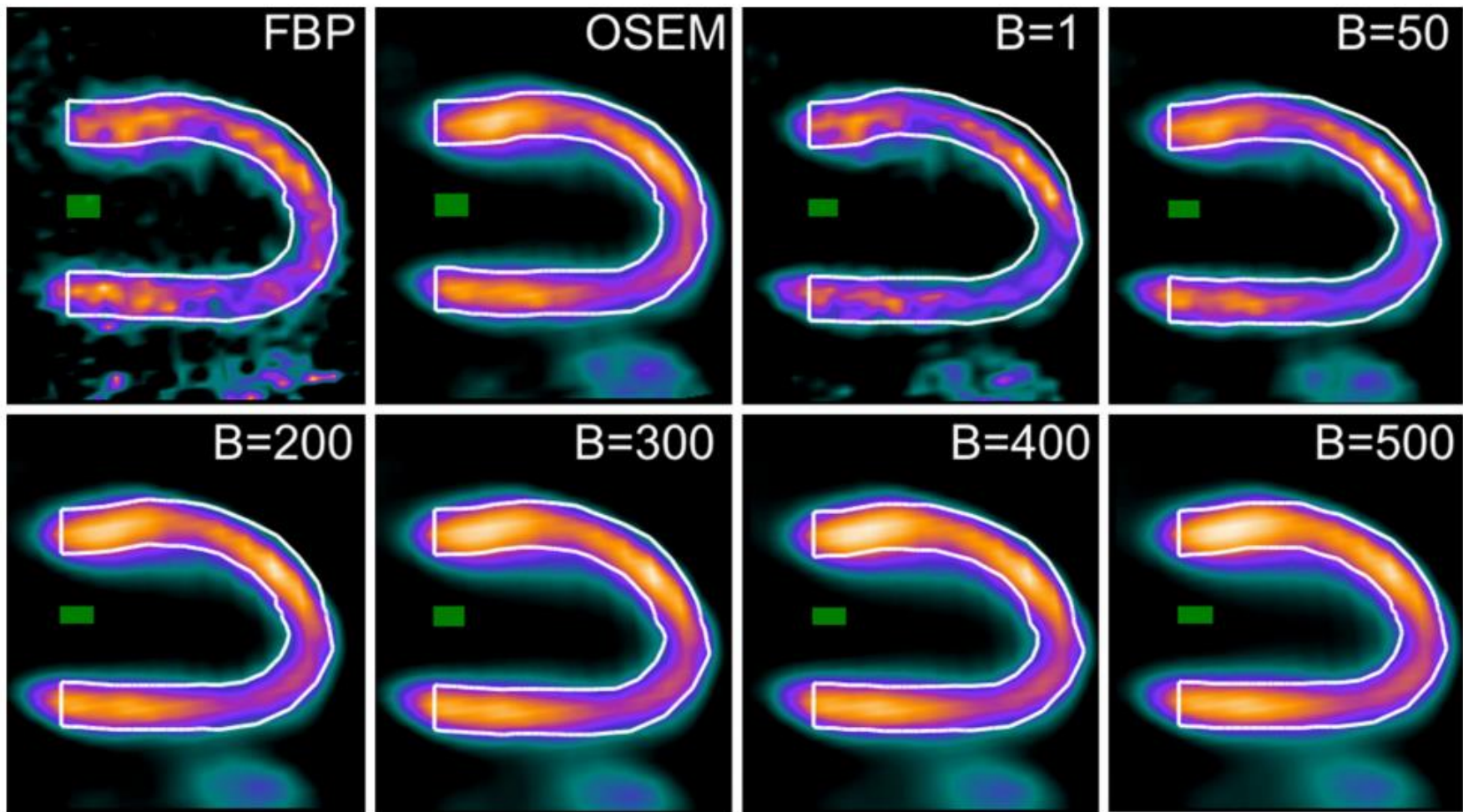
Figure 2. Automatic image segmentation of [13N]-NH3 images showing successful segmentation for all reconstructions. An automatically drawn 1 × 1 × 1 cm volume at the mitral valve plane represents the position of the left ventricular image-derived input function (IDIF). Images are displayed on the same window width and level.

Teoh, E. J., et al. Phantom and Clinical Evaluation of the Bayesian Penalized Likelihood Reconstruction Algorithm Q.Clear on an LYSO PET/CT System. *J. Nucl. Med.* **56**, 1447–1452 (2015).

Deep Learning for PET Recon

Deep Learning for PET Image Reconstruction

Andrew J. Reader, Guillaume Corda, Abolfazl Mehranian, Casper da Costa-Luis, Sam Ellis and Julia A. Schnabel, *IEEE Senior Member*
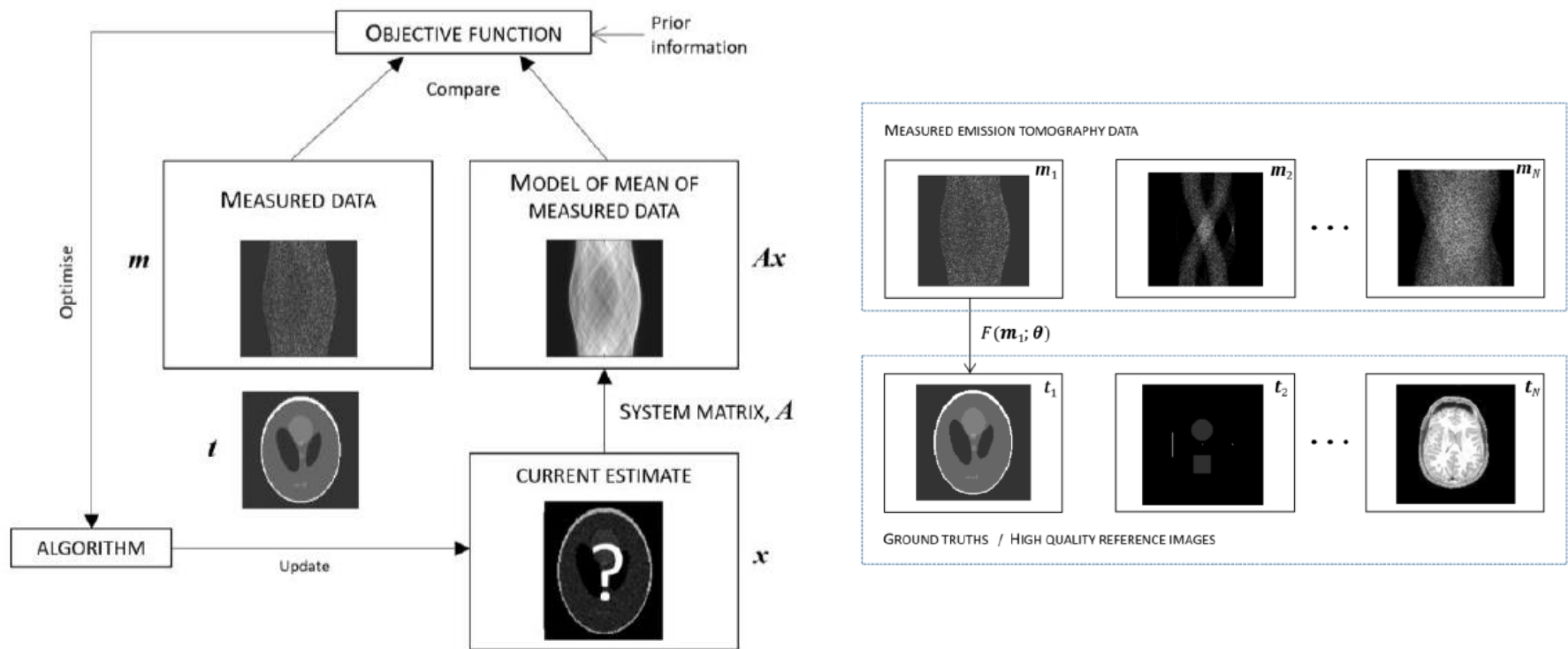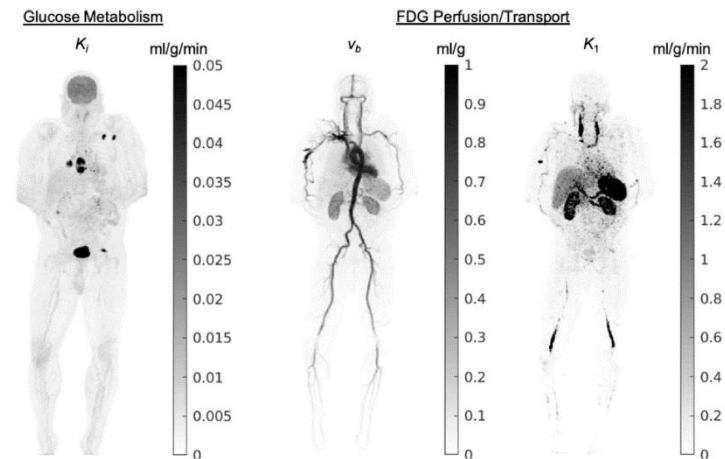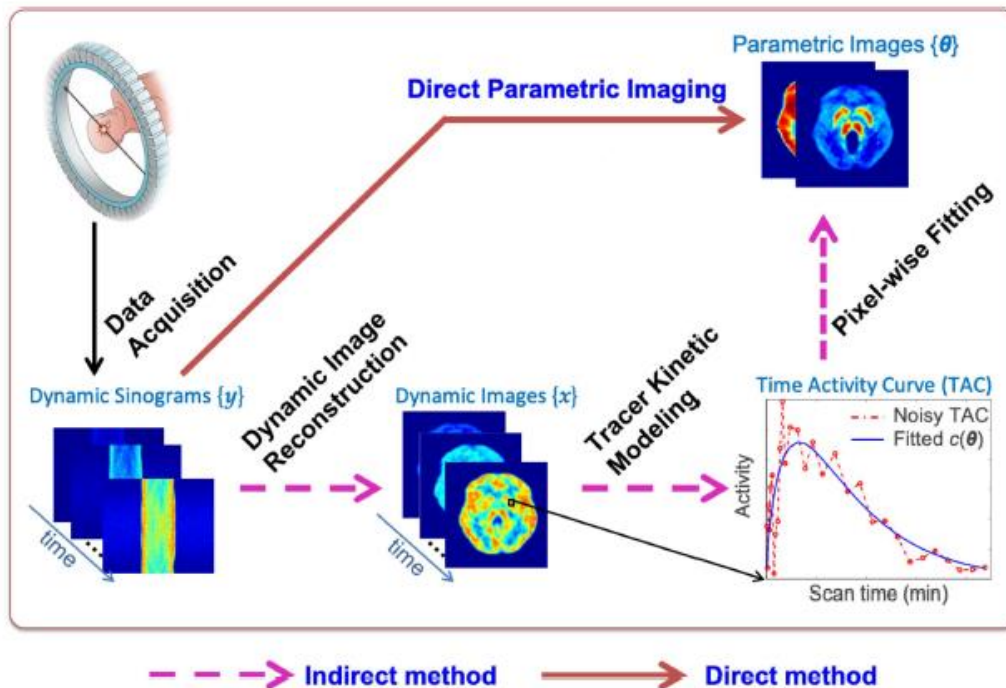
Parametric Image Reconstruction


PET Parametric Imaging: Past, Present, and Future

Guobao Wang, *Senior Member, IEEE*, Arman Rahmim, *Senior Member, IEEE*, and Roger N. Gunn

Reconless Reconstruction?

Ultrafast timing enables reconstruction-free positron emission imaging

Sun Il Kwon[1,5], Ryosuke Ota[2,5], Eric Berg[1,5], Fumio Hashimoto[2], Kyohei Nakajima[3], Izumi Ogawa[3], Yoichi Tamagawa[3], Tomohide Omura[2], Tomoyuki Hasegawa[4] and Simon R. Cherry[1]
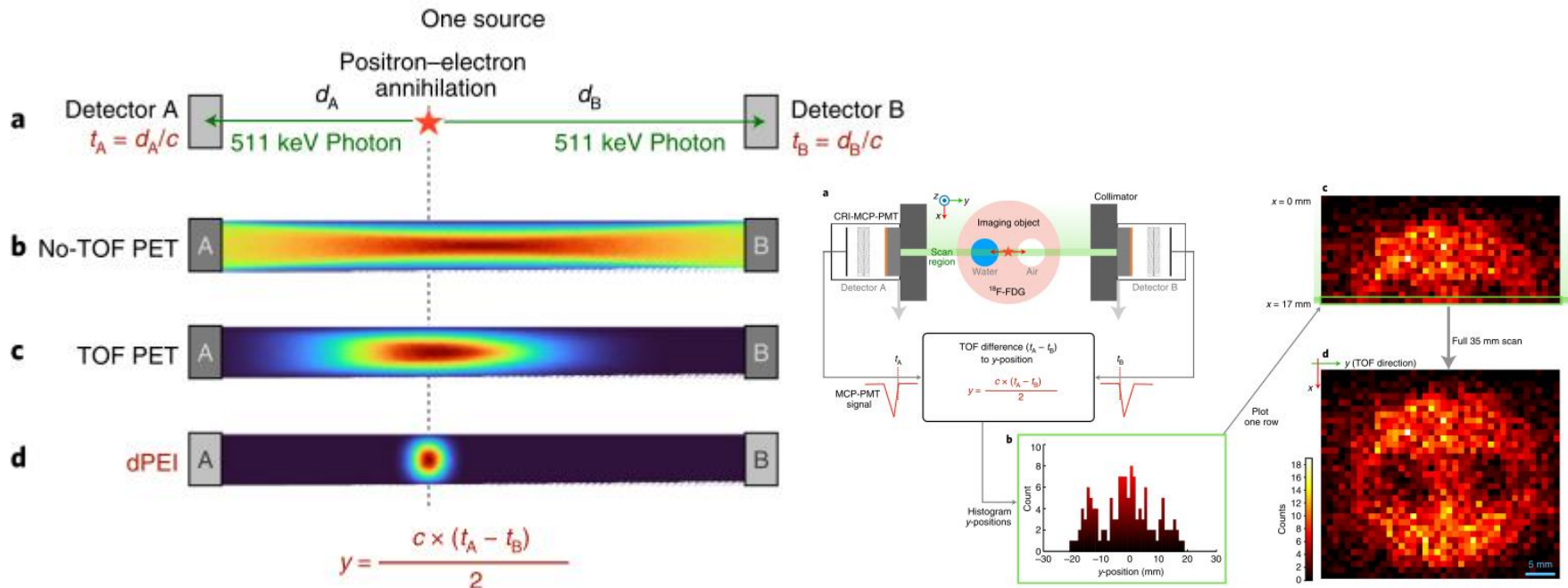
Possible Topics for Project Work / Master's Thesis

Currently available:
- ❑ GATE simulations with several PET systems
- ❑ Evaluation of open source reconstruction software
- ❑ PET flow phantom construction and evaluation
- ❑ Application of deep learning for PET and MRI image segmentation (*and other*)
- ❑ Establishing a software pipeline for ASL data processing

Past / ongoing projects:
- ❑ Development of pseudo-CT for PET/MR and MR-RT
- ❑ PET system evaluation (clinical and preclinical) using standard and custom phantom measurements
- ❑ Application of deep learning for image analysis
- ❑ QC tools and software for evaluation of imaging devices

Interested?

❑Contact: Jarmo Teuho (jarmo.teuho@tyks.fi or jatateu@utu.fi)

❑We got an large (and fun) group for you to work with, participate and support you during your thesis work!

❑Understanding on programming with Python, MATLAB, etc. is preferred in addition to theoretical background

❑See pages at:
  ▪https://sites.utu.fi/instrimpro/en/

Thank You!
(Time for Discussion)