UFYS2010: Radio astronomy instrumentation and interferometry

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Some of the figures are from Wilson, Rohlfs, Hüttemeister: 'Tools of Radio astronomy'

Outline

Recap from lecture 1

The ionospheric plasma attenuates radio radiation below the *plasma frequency*

$$\frac{\nu_\mathrm{p}}{\mathrm{kHz}} = 8.97\sqrt{\frac{N_\mathrm{e}}{\mathrm{cm}^{-3}}}. \tag{1}$$

Total flux density S_ν:

$$S_\nu = \int_{\Omega_\mathrm{s}} I_\nu(\theta,\phi)\cos\theta \, \mathrm{d}\Omega \tag{2}$$

The unit of S_ν is $\mathrm{W\,m^{-2}\,Hz^{-1}}$.

$$1\,\mathrm{Jy} = 10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}} \tag{3}$$

$$1\,\mathrm{SFU} = 10^{-22}\,\mathrm{W\,m^{-2}\,Hz^{-1}} \tag{4}$$

Radiative transfer

How intensity is changing if the properties of the medium are changed?

The intensity I_ν will change as a function of distance only if radiation is absorbed or emitted.

The change along the line of shight s can be expressed as follows:

$$\mathrm{d}I_{\nu-} = -\kappa_\nu I_\nu \,\mathrm{d}s, \tag{5}$$

$$\mathrm{d}I_{\nu+} = \epsilon_\nu \,\mathrm{d}s, \tag{6}$$

where $\mathrm{d}I_{\nu-}$ and $\mathrm{d}I_{\nu+}$ are the loss and gain terms respectively. The *equation of transfer* is then

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu I_\nu + \epsilon_\nu \tag{7}$$

Special cases of radiative transfer I

There are limiting cases for which the solution of the previous differential equation are especially simple:

1. Only emission i.e. $\kappa_\nu = 0$

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = \epsilon_\nu, \tag{8}$$

$$I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} \epsilon_\nu(s)\,\mathrm{d}s. \tag{9}$$

Special cases of radiative transfer II

2. Only absorption i.e. $\epsilon_\nu = 0$

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu I_\nu, \tag{10}$$

$$I_\nu(s) = I_\nu(s_0) exp\left(-\int_{s_0}^{s} \kappa_\nu(s)\,\mathrm{d}s\right). \tag{11}$$

Special cases of radiative transfer III

3. Full thermodynamical equilibrium (TE) of the radiation with surroundings

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = 0, \tag{12}$$

$$I_\nu(s) = B_\nu(T) = \epsilon_\nu/\kappa_\nu \tag{13}$$

The distribution is described by the Planck law:

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1}. \tag{14}$$

Full thermodynamic equilibrium will be realized only in very special circumstances such as in a black enclosure or, say, in stellar interiors.

Special cases of radiative transfer IV

4. Local thermodynamical equilibrium (LTE)

Often the Kirchoff's law

$$\frac{\epsilon_\nu}{\kappa_\nu} = B_\nu(T) \tag{15}$$

is locally applicable independentof the material as is case with complete thermodynamical equilibrium.

> "For a body of any arbitrary material, emitting and absorbing thermal electromagnetic radiation at every wavelength in thermodynamic equilibrium, the ratio of its emissive power to its dimensionless coefficient of absorption is equal to a universal function only of radiative wavelength and temperature, the perfect black-body emissive power."

In these cases I_ν will differ from $B_\nu(T)$.

Optical depth or opacity τ

When *optical depth* τ_ν is defined as

$$\tau_\nu(s) = \int_{s_0}^{s} \kappa_\nu(s)\,\mathrm{d}s, \qquad (16)$$

and it is further assumed that the medium is isothermal, we get

$$I_\nu(s) = I_\nu(0)e^{-\tau_\nu(s)} + B_\nu(T)\left(1 - e^{-\tau_\nu(s)}\right), \qquad (17)$$

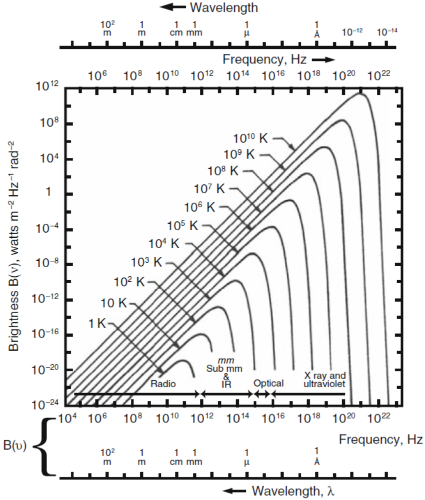
e.g. the medium decreases the observed intensity but also radiates black body radiation proportionally. If optical depth is very large, the observed intensity is totally black body radiation:

$$I_\nu = B_\nu(t). \qquad (18)$$

The spectrum of the radiation of a black body in thermodynamic equilibrium is given by the Planck law:

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}, \qquad (19)$$

which gives the power per unit bandwidth $(\frac{\mathrm{W}}{\mathrm{Hz}})$.



The Stefan-Boltzmann radiation law

When the Planck law is integrated over frequency, we get the *total brightness of black body* or *Stefan-Boltzmann radiation law*:

$$B(T) = \sigma T^4, \quad (20)$$

where

$$\sigma = \frac{2\pi^4 k^4}{15 c^2 h^3}. \quad (21)$$

The **radiation maximum** of Stefan-Boltzmann radiation law is at the frequency

$$\nu_{\max}[\mathrm{GHz}] = 58.789\, T[\mathrm{K}], \quad (22)$$

and at the wavelength of

$$\lambda_{\max}[\mathrm{mm}]\, T[\mathrm{K}] = 2.8978. \quad (23)$$

These both are called as *Wien's displacement law*.

Planckin law approximations, Rayleigh-Jeans law

The Planck law can be approximated by simpler expressions at the frequency extremes.

Rayleigh-Jeans law, $h\nu \ll kT$:

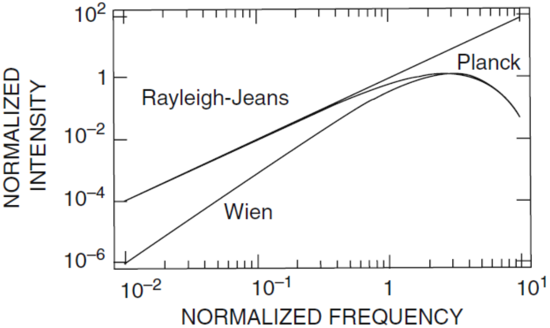
$$B_{\mathrm{RJ}}(\nu, T) = \frac{2\nu^2}{c^2} kT = \frac{2kT}{\lambda^2} \tag{24}$$

Widely used in radio astronomy. Holds when $\nu \ll 20.84\,T$ where ν is in gigahertz and T in Kelvins.

Wien's law, $h\nu \gg kT$:

$$B_\mathrm{W}(\nu, T) = \frac{2h\nu^3}{c^2} e^{-h\nu/kT} \tag{25}$$

Holds in visible and ultraviolet, not in radio.



Brightness temperature

Rayleigh-Jeans law implies that the brightness and the temperature of the black body that emits this radiation are strictly proportional. Because of this, the brightness of an extended radio source is measured using its *brightness temperature* T_b.

$$T_b = \frac{\lambda^2}{2k} I_\nu \tag{26}$$

Because

$$S_\nu = I_\nu \Delta\Omega, \tag{27}$$

the total flux density radiated by a black body is

$$S_\nu = \frac{2k}{\lambda^2} T_b \Delta\Omega \tag{28}$$

The concept of brightness temperature is used for convenience also in cases where the radiation is not caused by a black body. Then it is the *equivalent* brightness temperature that corresponds to a black body in the given temperature.

Total flux density and brightness temperature

For a Gaussian source with a brightness temperature T_b the total flux density is

$$S_\nu = \frac{2.65\, T_b\, \theta^2}{\lambda^2}, \tag{29}$$

where S_ν is in Jansky, θ in arcminutes, and λ in cm.

Brightness temperature and opacity

When a source is observed through an absorbing medium, e.g. the atmosphere, the observed brightness temperature is changed from the original $T_b(0)$ as a function of the attenuation and the temperature of the physical medium. The medium absorbs the radiation but also radiates and increases the observed brightness temperature.

If the medium is isothermal, the observed brightness temperature $T_b(s)$ with medium thickness s is

$$T_b(s) = T_b(0)e^{-\tau_\nu(s)} + T(1 - e^{-\tau_\nu(s)}). \qquad (30)$$

If opacity is very small, the absorbation and radiation of the medium has no effet. However, if it is very large, the observed brightness temperature is the *physical temperature of the medium*.

Nyqvist theorem and noise temperature

Radio astronomy receivers and detectors measure a (very small) electric power. The relation between electric power and temperature makes a connection between the received electic power and source noise temperature and finally to total flux density.

A resistor, at a temperature of T, produces an electric power of P (electrons are moving due to the temperature i.e Johnson noise). This phenomenon is reciprocal, i.e. if electric power is dissipated by the resistor, its temperature is rising by T:

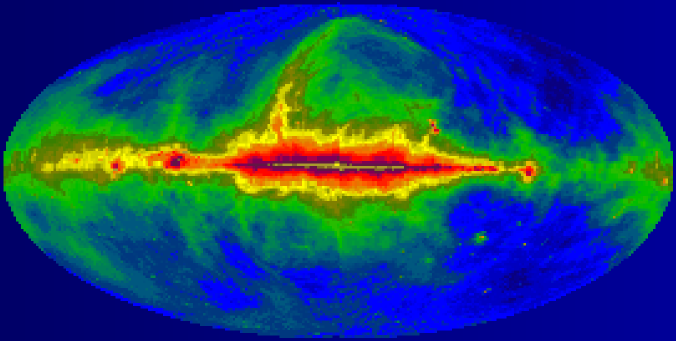
$$P_\nu = kT, \tag{31}$$

i.e. the total power in unit bandwidth, or the total power

$$P = kTB, \tag{32}$$

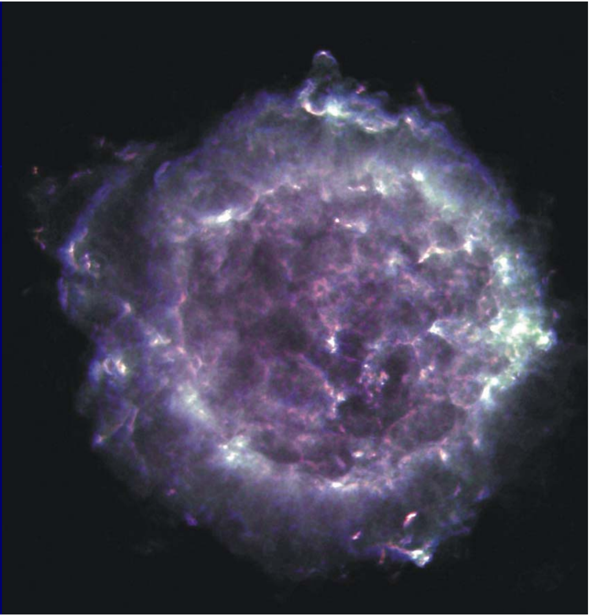
where k is the Boltzmann constant and B is bandwidth.

408 MHz continuum emission, galactic coordinates

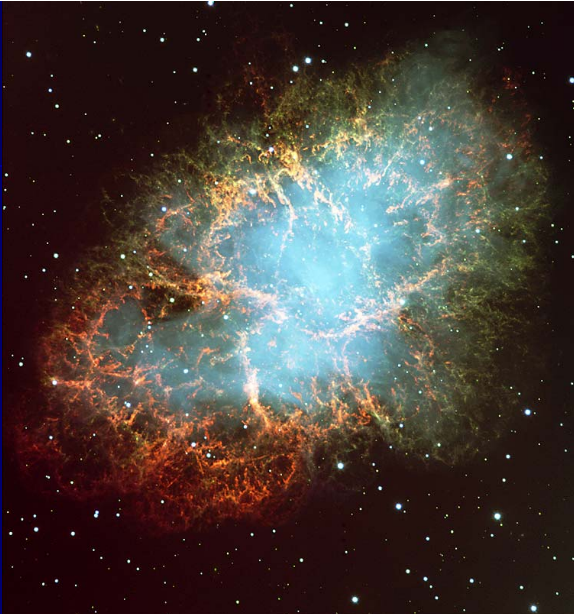


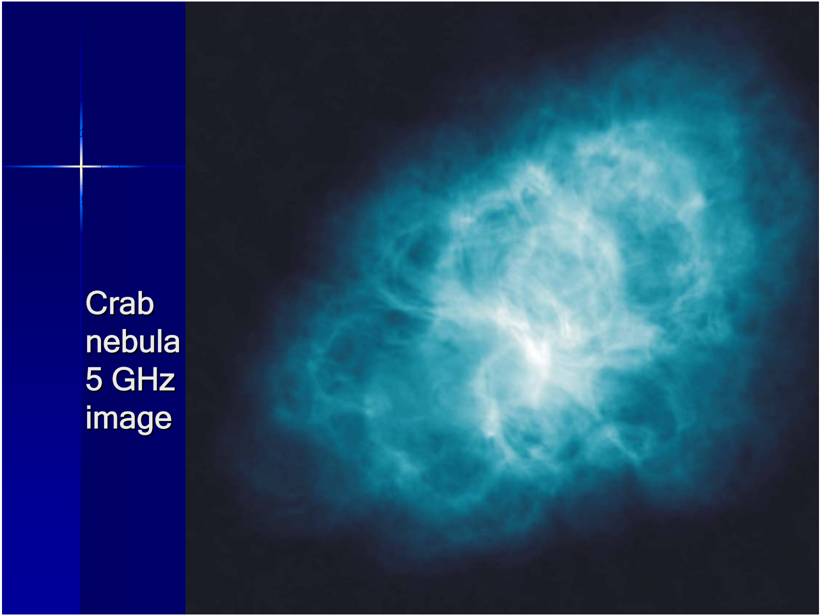

4.85 GHz
sky over
Green Bank

Cas A: supernova remnant at 1.4, 5, and 8 GHz

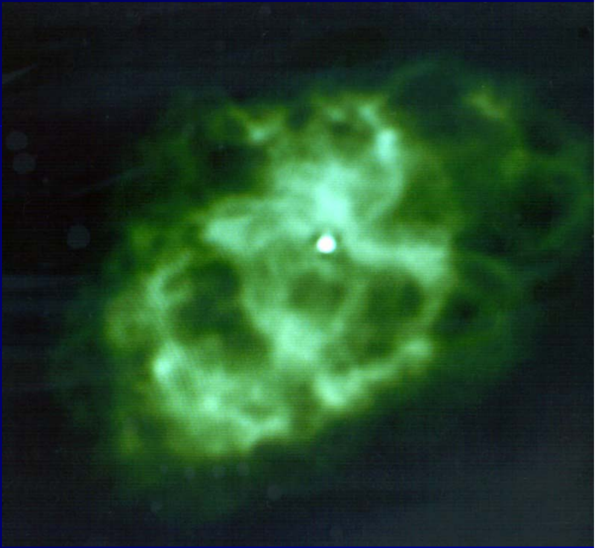


Crab Nebula
remnant of
1054 AD
supernova

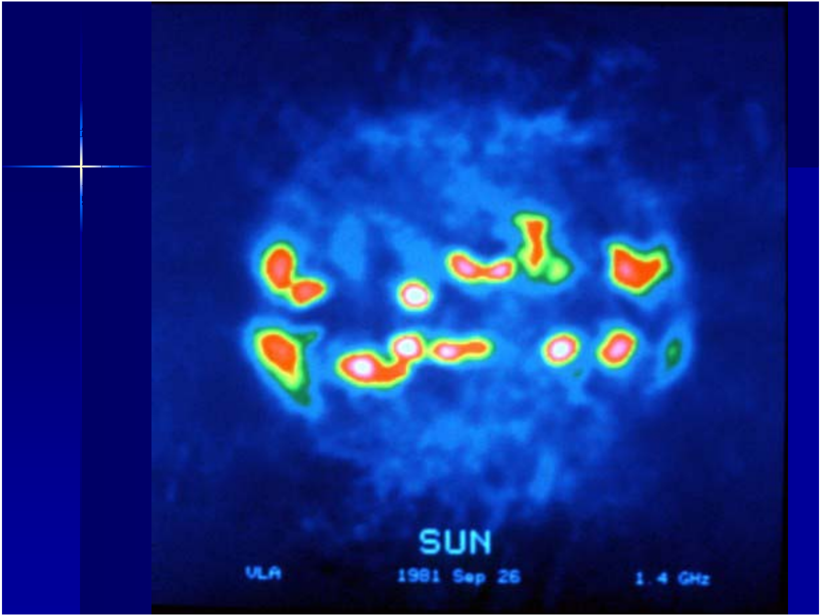



Crab
nebula
5 GHz
image

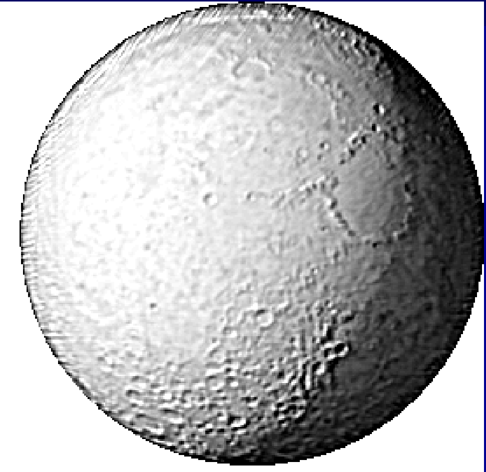
Crab nebula and pulsar at 327 MHz

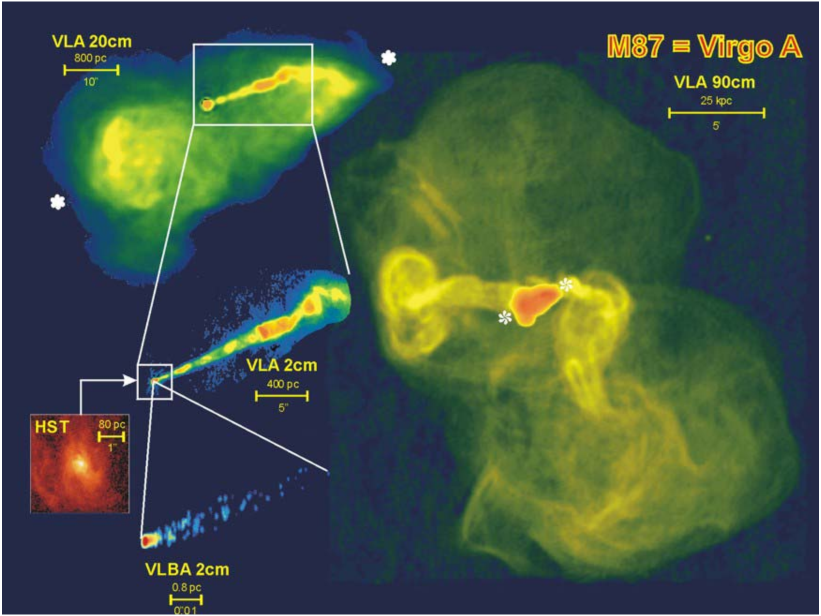


VLA (1 km D-configuration)


SUN
VLA
1981 Sep 26
1.4 GHz

850 micron thermal emission from the Moon, observed with SCUBA on the JCMT




M87 = Virgo A
VLA 90cm
25 kpc
5'
VLA 20cm
800 pc
10"
VLA 2cm
400 pc
5"
HST
80 pc
1"
VLBA 2cm
0.8 pc
0.01"