UFYS2010: Radio astronomy instrumentation and interferometry

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Some of the figures are from Wilson, Rohlfs, Hüttemeister: 'Tools of Radio astronomy'

Outline

Recap from lecture 5

Vector waves

Poincaré sphere

Stokes parameters

Rotation measure

Polarimetry

Radiometer sensitivity in precense of gain variations

The minimum detectable noise temperature of Dicke radiometer:

$$\Delta T_{min} = 2T_{sys}\sqrt{\frac{1}{\Delta\nu\tau} + \left(\frac{T_{sky} - T_{ref}}{T_{sys}} \cdot \frac{\Delta G}{G}\right)^2}, \qquad (1)$$

Gain variations are cancelled totally if

- the reference and antenna noise temperatures are equal and
- the switching frequency is suffciently high so that the gain can be considered nearly constant in the sky-reference cycle.

Pseudocorrelation radiometer

- It does not have $\sqrt{2}$ penalty in sensitivity because it observes the source continously.
- It observes also the reference continuously, even the fastest variations get cancelled.
- Like all switched radiometers, it must be *balanced*.

Vector waves

The electric field vector of a monochromatic electromagnetic plane wave is perpendicular to the direction of the field. The vector can be given as a sum of two orthogonal components:

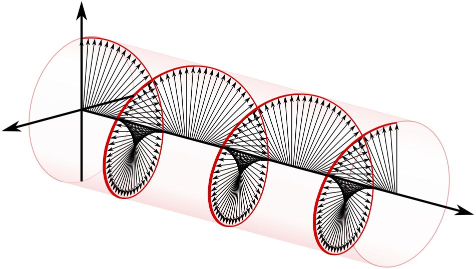
$$
\begin{aligned}
E_x &= E_1 \cos(kz - \omega t + \delta_1) \\
E_y &= E_2 \cos(kz - \omega t + \delta_2) \\
E_z &= 0,
\end{aligned} \tag{2}
$$

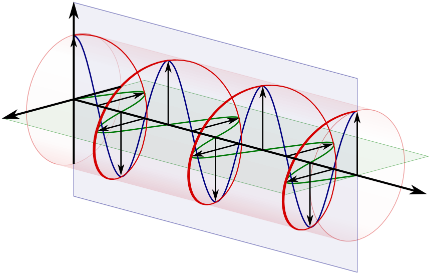
where $k = 2\pi/\lambda$ and $\omega = 2\pi\nu$. The equation (2) describes a helix on the surface of a cylinder.
The cross section is

$$
\left(\frac{E_x}{E_1}\right)^2 + \left(\frac{E_y}{E_2}\right)^2 - 2\frac{E_x}{E_1}\frac{E_y}{E_2}\cos\delta = \sin^2\delta, \tag{3}
$$

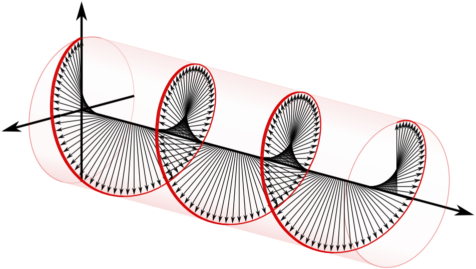
where $\delta = \delta_1 - \delta_2$.

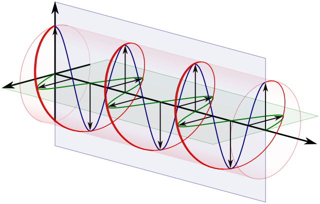
Circularly polarized wave (left-hand)



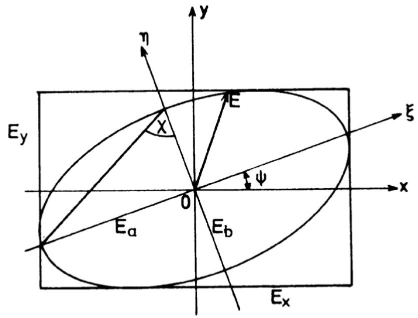


Circularly polarized wave (right-hand)





Polarization ellipse



Equation (3) defines an ellipse, therefore a general electromagnetic wave is *elliptically polarized* (including linear and circular). $\sin\delta$ determines the sense of rotation of the electric vector.

The orientation of the ellipse (3) is usually arbitrary. When a coordinate system is aligned with the major and minor axes of the ellipse, we getsi $(\tau = kz - \omega t)$

$$E_\xi = E_a \cos(\tau + \delta)$$
$$E_\eta = E_a \sin(\tau + \delta). \quad (4)$$

$$E_\xi = E_x \cos\Psi + E_y \sin\Psi$$
$$E_\eta = -E_x \sin\Psi + E_y \cos\Psi \quad (5)$$

defines the coordinate transformation.

Polarization parameters

The *total flux* or *Poynting flux* is defined as:

$$S_0 \equiv E_a^2 + E_b^2 = E_1^2 + E_2^2, \tag{6}$$

i.e. it can be expressed as a sum of squares of two arbitrary orthogonal field components.
The polarization state of an electromagnetic wave can be expressed *exactly* with the following parameters:

$$\frac{E_1}{E_2} = \tan\alpha \tag{7}$$

$$\tan 2\Psi = -\tan 2\alpha \cos\delta \tag{8}$$

$$\frac{E_a}{E_b} = \tan\chi \tag{9}$$

$$\sin 2\chi = \sin 2\alpha \sin\delta \tag{10}$$

Phase shift δ

The sign of δ and χ defines the polarization rotation direction:

- $\sin\delta > 0$ tai $\tan\chi > 0 \Rightarrow$ **righthanded polarizationo**
- $\sin\delta < 0$ tai $\tan\chi < 0 \Rightarrow$ **lefthanded polarization**

In a righthanded polarized wave, the field vector **E** rotates along the wave direction as right handed thread.

Further, if

$$\delta = \delta_1 - \delta_2 = m\pi, m = 0, \pm 1, \pm 2 \ldots \tag{11}$$

the polarization ellipse is reduced to a line and we have *linear polarization*.

Circular polarization

Another special case is when

$$E_1 = E_2 = E \tag{12}$$

and

$$\delta = \frac{\pi}{2}(1+m), m = 0, 1, \pm 2, \pm 3 \dots \tag{13}$$

In this case the equation of the polarization ellipse (3) is reduced to an equation of a circle:

$$E_x^2 + E_y^2 = E \tag{14}$$

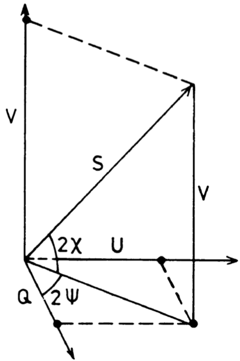
An elliptically polarized wave can be expressed as the sum of two orthogonal wave:

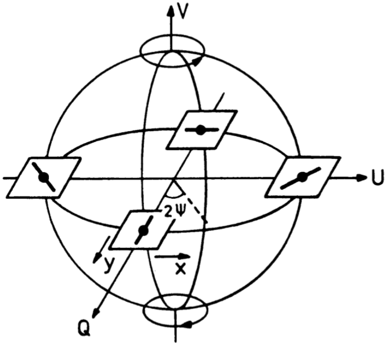
$$\begin{aligned} E_r &= \frac{1}{2}(E_a + E_b) \\ E_l &= \frac{1}{2}(E_a - E_b) \end{aligned} \tag{15}$$

Poynting flux can then be expressed as the sum of squares of these components:

$$S_0 = E^2 + E^2 = E^2 + E^2 \tag{16}$$

Poincaré sphere and Stokes parameters





If 2Ψ is interpreted as longitude and 2χ as latitude on a sphere with a radius of S_0, all polarization states can be expressed on a surface of a sphere.

The positions on equator correspond to linear polarization, north pole right hand circular and south pole left hand circular polarization. The axes correspond to *Stokes parameters*.

Stokes parameters I

Using the Poincaré sphere, *Stokes parameters* can be defined conveniently:

$$\begin{aligned} S_0 &= I = E_a^2 + E_b^2 \\ S_1 &= Q = S_0 \cos 2\chi \cos 2\Psi \\ S_2 &= U = S_0 \cos 2\chi \sin 2\Psi \\ S_3 &= V = S_0 \sin 2\chi \end{aligned} \tag{17}$$

Only three of the parameters are independent, one of them can be expressed using the three others:

$$\begin{aligned} S_0^2 &= S_1^2 + S_2^2 + S_3^2 \\ I^2 &= Q^2 + U^2 + V^2 \end{aligned} \tag{18}$$

Stokes parameters II

The previous definitions can be used to express the polarization state of a wave exactly but they do not correspond directly to observable quantities. By expressing the parameters using field vectors and their phase shift, we get:

$$\begin{aligned} S_0 &= I = E_1^2 + E_2^2 \\ S_1 &= Q = E_1^2 - E_2^2 \\ S_2 &= U = 2E_1E_2\cos\delta \\ S_3 &= V = 2E_1E_2\sin\delta \end{aligned} \tag{19}$$

Special cases:

- Right-handed circular polarization: $I = V = S$ and $Q = U = 0$
- Left-handed circular polarization: $I = S$, $V = -S$ and $Q = U = 0$
- Linear polarization: $I = S$, $V = 0$, $Q = I\cos 2\Psi$ and $U = I\sin 2\Psi$

Stokes parameters for a quasi-monochromatic wave

When the wave is not monochromatic but $\Delta\nu/\nu \ll 1$, the Stokes parameters can be expressed as the time average of the product of the field components $\langle E^2(t)\rangle = \langle E(t)\cdot E^*(t)\rangle$:

$$
\begin{aligned}
S_0 &= I = \langle a_1^2\rangle + \langle a_2^2\rangle \\
S_1 &= Q = \langle a_1^2\rangle - \langle a_2^2\rangle \\
S_2 &= U = \langle 2a_1 a_2 \cos\delta\rangle \\
S_3 &= V = \langle 2a_1 a_2 \sin\delta\rangle
\end{aligned}
\tag{20}
$$

Degree of polarization p

The quasimonochromatic wave is not necessarily fully polarized, then

$$\begin{aligned} S_0^2 &\geq S_1^2 + S_2^2 + S_3^2 \\ I^2 &\geq Q^2 + U^2 + V^2. \end{aligned} \tag{21}$$

Degree of polarization is defined as

$$p = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \tag{22}$$

The Stokes parameters of a general (quasimonochromatic) wave are the sum of Stokes parameters of the individual waves.

Faraday polarization rotation and rotation measure

Polarimetric observations give information of the magnetic field strength and direction and also of electron (charge) density.
This is due to Faraday effect where the electric vector polarization angle (EVPA) of a linearly polarized wave rotates when it travels trough a medium with plasma and magnetic field:

$$\chi^{\mathrm{lin}}(\lambda) = \chi_0^{\mathrm{lin}} + \mathrm{RM} \cdot \lambda^2, \tag{23}$$

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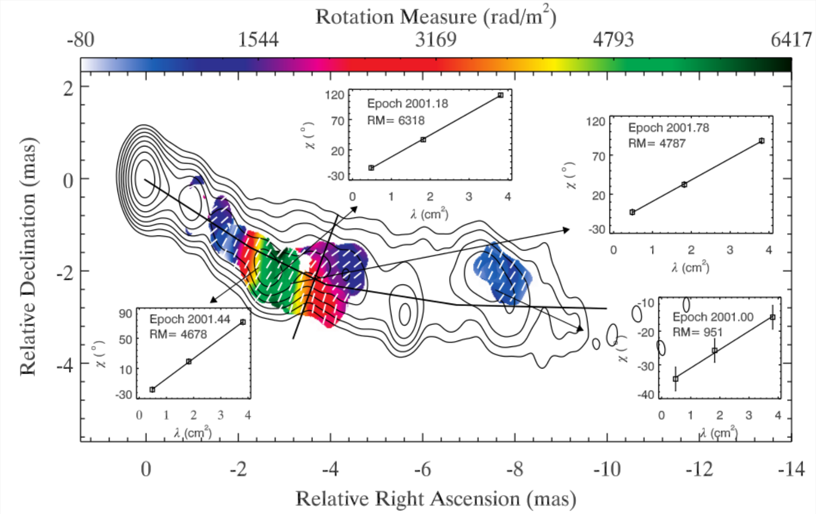
$$\mathrm{RM} = 0.81 \int_{\mathrm{source}}^{\mathrm{observer}} n_e \overrightarrow{B} \cdot \mathrm{d}\,\overrightarrow{l}\,. \tag{24}$$

χ_0^{lin} is the original EVPA, RM is *rotation measure*, n_e is electron density (cm^{-3}, B is magnetic field strength (μG), and l is travelled distance in parsec.

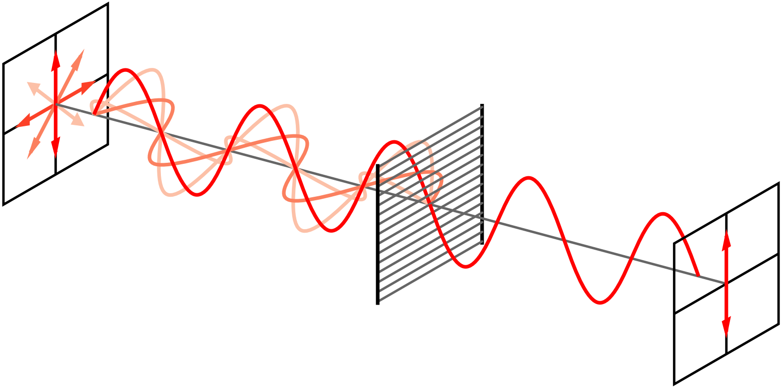
(Rohlfs et al. book uses χ in different context, because of this χ^{lin} is used here for EVPA)

Rotation measure in 3C 120

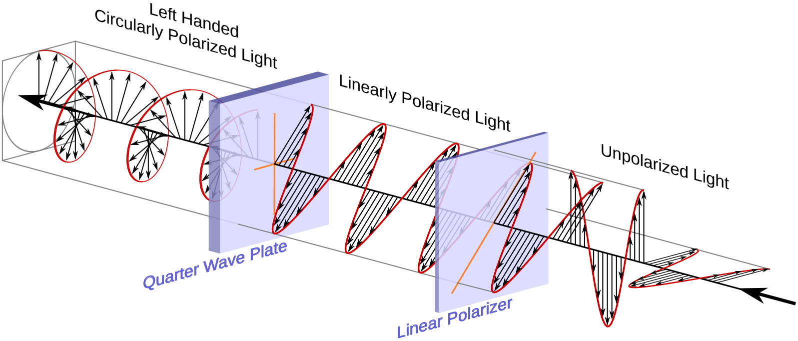


Gómez et al. ApJ 681: L69–L72

Wire grid polarizer (linear)



Circular polarizer ($\lambda/4$ plate)



$\lambda/2$ plate