UFYS2010: Radio astronomy instrumentation and interferometry

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Spring 2013

Based partly on 'Essential radio astronomy' from http://www.cv.nrao.edu/course/astr534/Interferometers2.html and http://www.cv.nrao.edu/course/astr534/Interferometers2.html by J. J. Condon and S. M. Ransom.

Outline

Recap from lecture 7

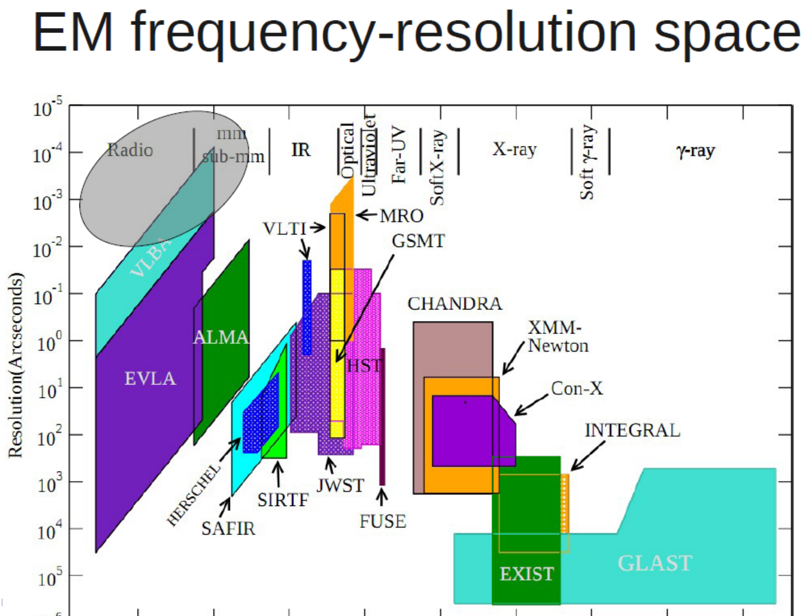
Complex correlator

Effect of finite observing bandwidth

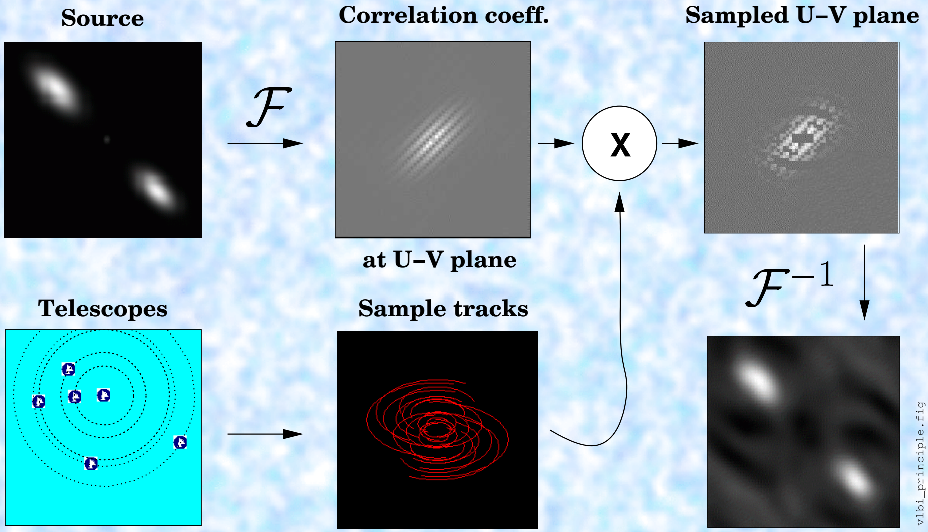
Delay compensation

Limits of field of view

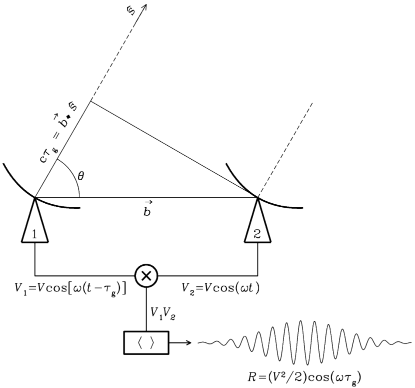
Recap: Resolution



VLBI (Very Long Baseline Interferometry)



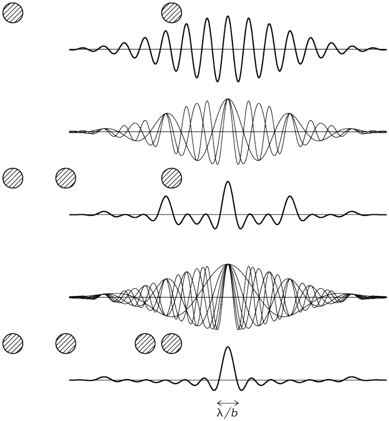
Recap: Correlation



After multiplication the product of the antenna voltages is averaged with a timescale of typically seconds, so that the term $\cos(2\omega t - \omega\tau_g)$ averages out.

$$V_1 V_2 = V^2 \cos(\omega t)\cos[\omega(t-\tau_g)] = \left(\frac{V^2}{2}\right)[\cos(2\omega t - \omega\tau_g) + \cos(\omega\tau_g)] \tag{1}$$

Recap: Fourier spatial frequencies and synthesized beam



An interferometer with N antennas contains $N(N-1)/2$ pairs of antennas i.e. baselines
These cases have

- one baseline b
- three baselines b, $b/3$, $2b/3$
- six baselines b, $b/6$, $2b/6$, $3b/6$, $4b/6$, $5b/6$

When the number of **unique** baselines are increased, the synthesized beam approaches Gaussian with angular resolution $\approx \lambda/b$.

Extended Sources and the Complex Correlator

Point sources can be observed with a simple interferometer consisting of only one *cosine* correlator:

$$R_{\rm c} = (V^2/2)\cos(\omega\tau_{\rm g}). \quad (2)$$

But interferometers are used mainly for *imaging* structures, not only point sources. The sky brightness distribution $I_\nu(\hat{s})$ around frequency $\nu = \omega/(2\pi)$ can be treated as a collection of point sources:

$$R_{\rm c} = \int I_\nu(\hat{s})\cos(2\pi\nu\vec{b}\cdot\hat{s}/c)\,{\rm d}\Omega = \int I_\nu(\hat{s})\cos(2\pi\vec{b}\cdot\hat{s}/\lambda)\,{\rm d}\Omega\ . \quad (3)$$

A problem:

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A problem:

Cosine function is *even* and it is sensitive only to *even* i.e. inversion symmetric part $I_{\rm E}$ of the brightness distribution.

Even and odd, symmetric and antisymmeric distributions

How to include the antisymmetric part?

Even and odd, symmetric and antisymmeric distributions

How to include the antisymmetric part?

Simple by summing up symmetric and antisymmetric distributions: $I = I_{\mathrm{E}} + I_{\mathrm{O}}$.

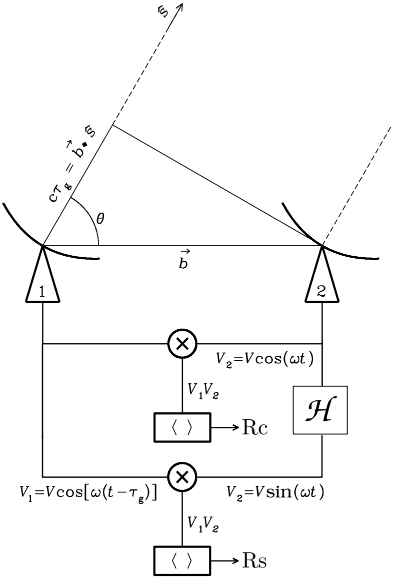
A convenient antisymmetric function is of course sine. Response for a point source is:

$$R_{\mathrm{s}} = (V^2/2)\sin(\omega\tau_{\mathrm{g}}) \tag{4}$$

and for extended emission:

$$R_{\mathrm{s}} = \int I_\nu(\hat{s})\sin(2\pi\vec{b}\cdot\hat{s}/\lambda)\,\mathrm{d}\Omega \ . \tag{5}$$

How to implement a sine correlator?



The sine part of the correlator can be implemented simply by introducing 90° phase shift to the second branch of the correlator. The 90° phase shift is mathematically the *Hilbert transform* $\mathcal{H}$ of the signal.

Complex correlator

Sine/cosine correlator is called a *complex* correlator because the output, *visibility* is a complex number, generally

$$e^{i\phi} = \cos(\phi) + i\sin(\phi)\ . \tag{6}$$

Complex visibility is defined as

$$V \equiv R_{\rm c} - iR_{\rm s} \tag{7}$$

and it can be written in the form

$$V = Ae^{-i\phi}\ , \tag{8}$$

where

$$A = (R_{\rm c}^2 + R_{\rm s}^2)^{1/2} \tag{9}$$

is the visibility amplitude and

$$\phi = \tan^{-1}(R_{\rm s}/R_{\rm c}) \tag{10}$$

is the visibility phase.

Complex correlator response to an extended emission

Complex visibility is defined as

$$V \equiv R_{\mathrm{c}} - iR_{\mathrm{s}} \tag{11}$$

and it can be written in the form

$$V = Ae^{-i\phi} \ . \tag{12}$$

The response of a two element interferometer with a complex correlator is

$$V_\nu = \int I_\nu(\hat{s}) \exp(-i2\pi\vec{b}\cdot\hat{s}/\lambda) \ \mathrm{d}\Omega \ . \tag{13}$$

Effect of finite observing bandwidth I

$$V_\nu = \int I_\nu(\hat{s}) \exp(-i2\pi \vec{b} \cdot \hat{s}/\lambda) \, \mathrm{d}\Omega \, . \tag{14}$$

In the equation above, we have assumed that the emission bandwidth is infinitely small (δ-function). If the source emission is constant over a small but finite frequency range $\Delta\nu$ centered on frequency ν_c, we get

$$V = \int \left[(\Delta\nu)^{-1} \int_{\nu_\mathrm{c}-\Delta\nu/2}^{\nu_\mathrm{c}+\Delta\nu/2} I_\nu(\hat{s}) \exp(-i2\pi \vec{b} \cdot \hat{s}/\lambda) \, \mathrm{d}\nu \right] \mathrm{d}\Omega \tag{15}$$

or in terms of geometric delay

$$V = \int \left[(\Delta\nu)^{-1} \int_{\nu_\mathrm{c}-\Delta\nu/2}^{\nu_\mathrm{c}+\Delta\nu/2} I_\nu(\hat{s}) \exp(-i2\pi \nu \tau_\mathrm{g}) \, \mathrm{d}\nu \right] \mathrm{d}\Omega \, . \tag{16}$$

Effect of finite observing bandwidth II

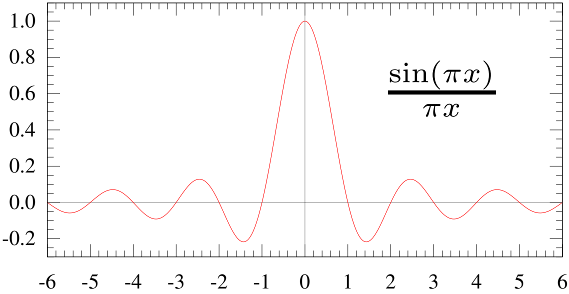
$$V = \int \left[(\Delta\nu)^{-1} \int_{\nu_c - \Delta\nu/2}^{\nu_c + \Delta\nu/2} I_\nu(\hat{s}) \exp(-i 2\pi\nu\tau_g) \, d\nu \right] d\Omega \, . \qquad (17)$$

Effect of finite observing bandwidth II

$$V = \int \left[(\Delta\nu)^{-1} \int_{\nu_c - \Delta\nu/2}^{\nu_c + \Delta\nu/2} I_\nu(\hat{s}) \exp(-i2\pi\nu\tau_g) \, d\nu \right] d\Omega \,. \qquad (17)$$

The integral in the brackets is a Fourier transform of rectangle which is the *sinc* function, so the integral can be written as

$$V = \int I_\nu(\hat{s}) \mathrm{sinc}(\Delta\nu\tau_g) \exp(-i2\pi\nu_c\tau_g) \, d\Omega \,. \qquad (18)$$



Delay compensation

$$V = \int I_\nu(\hat{s}) \mathrm{sinc}(\Delta\nu\tau_\mathrm{g}) \exp(-i2\pi\nu_\mathrm{c}\tau_\mathrm{g}) \, \mathrm{d}\Omega \ . \qquad (19)$$

What does this mean?

Delay compensation

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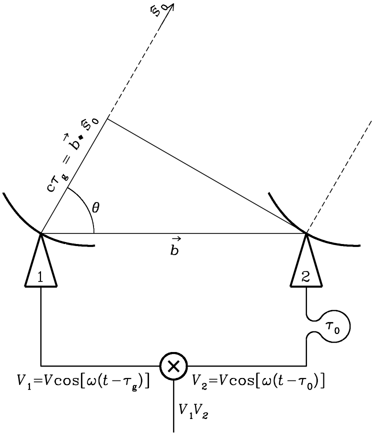
What does this mean?



For a finite bandwidth and non-zero delay, the fringe amplitude is attenuated by the factor $\mathrm{sinc}(\Delta\nu\tau_\mathrm{g})$. This can be compensated in any *one* direction $\hat{s}_0$, that is called the *delay center* by using a compensating delay $\tau_0 \approx \tau_\mathrm{g}$ (usually digital).

Field of view limit due to finite bandwidth I

Because the delay varies with direction, delay compensation works only within a certain angular distance from the delay center where $|\tau_0 - \tau_{\rm g}| \ll (\Delta\nu)^{-1}$.

The angular radius $\Delta\theta$ of the usable field-of-view depends on the variation of $\tau_{\rm g}$ as a function of direction $\hat{s}_0$.

Because

$$c\tau_{\rm g} = \vec{b} \cdot \vec{s} = b\cos\theta \ , |c\Delta\tau_{\rm g}| = b\sin\theta\Delta\theta \ , \qquad (20)$$

requiring that $\Delta\nu\Delta\tau_{\rm g} \ll 1$ implies

$$\Delta\nu(b\sin\theta)\Delta\theta/c \ll 1 \ . \qquad (21)$$

Further as $\lambda\nu = c$ and syntethized beamwidth is $\theta_{\rm s} \approx \lambda/(b\sin\theta)$ we get the requirement

$$\Delta\theta\Delta\nu \ll \theta_{\rm s}\nu \ . \qquad (22)$$

Field of view limit due to finite bandwidth II

$$\frac{\Delta\theta\Delta\nu}{\nu} \ll \theta_{\mathrm{s}} \qquad (23)$$

When the offsets from the delay centre are larger, bandwidth smearing broadens the synthesized beam by convolution with a rectangle of angular width $\Delta\theta\Delta\nu/\nu$.

So, what can be done for wide-field imaging?

Field of view limit due to finite bandwidth II

$$\frac{\Delta\theta\Delta\nu}{\nu} \ll \theta_{\rm s} \qquad (23)$$

When the offsets from the delay centre are larger, bandwidth smearing broadens the synthesized beam by convolution with a rectangle of angular width $\Delta\theta\Delta\nu/\nu$.

So, what can be done for wide-field imaging?

Splitting the bandwidth into narrow chunks.

Field of view limit due to finite bandwidth, example

Example:
VLA B-configuration, $b \approx 10$ km, $\lambda = 20$ cm i.e. $\nu = 1.5$ GHz the field of view is limited to $\theta_\mathrm{s} \approx [(0.2\ \mathrm{m})/(10^4\ \mathrm{m})]\ \mathrm{rad} \approx 4\ \mathrm{arcsec}$.

But what if we need to image an area of $\Delta\theta = 15\ \mathrm{arcmin} = 900\ \mathrm{arcsec}$ that corresponds to the single antenna primry beam?

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But what if we need to image an area of $\Delta\theta = 15$ arcmin $= 900$ arcsec that corresponds to the single antenna primry beam?

900 arcsec field of view can be obtained by using single channel bandwidth of

$$\Delta\nu \ll \frac{\nu\theta_s}{\Delta\theta} = \frac{1.5\times10^9 \text{ Hz}\times4 \text{ arcsec}}{900 \text{ arcsec}} \approx 7 \text{ MHz}$$

Time smearing

Like finite bandwith, finite correlator integration time smears images with large fields. This is because Earth's rotation moves the source position in the frame of the interferometer.
This should be kept much smaller than the synthesized beam $\theta_{\rm s} \approx \lambda/b$. E.g. if tracking the north celestial pole, source $\Delta\theta$ away will move at an angular rate of $2\pi\Delta\theta/P$,
$P \approx 23^{\rm h}56^{\rm m}04^{\rm s} \approx 86164\ {\rm s}$.
If correlator averaging time is long compared this apparent movement, the synthesized beam will broaden tangentially. To minimize this

$$\Delta\theta\Delta t \ll \frac{\theta_{\rm s} P}{2\pi} \approx \theta_{\rm s} \times 1.37 \times 10^4\ {\rm s} \quad (24)$$

Time smearing, example

$$\Delta\theta\Delta t \ll \frac{\theta_\mathrm{s} P}{2\pi} \approx \theta_\mathrm{s} \times 1.37 \times 10^4 \ \mathrm{s} \qquad (25)$$

Example

Like in the previous example, if we want to image an area of $\Delta\theta = 900\ \mathrm{arcsec}$ when $\theta_\mathrm{s} = 4\ \mathrm{arcsec}$, the averaging time should be

$$\Delta t \ll \frac{\theta_\mathrm{s}}{\Delta\theta} \times 1.37 \times 10^4 \ \mathrm{s} = \frac{4\ \mathrm{arcsec}}{900\ \mathrm{arcsec}} \times 1.37 \times 10^4 \ \mathrm{s} \approx 60\ \mathrm{s}$$

Note that we are considering here the **correlator integration time**, i.e. the integration time of a single visibility data point. The full image integration time can be practically anything: minutes, hours and even longer if the source structure does not change.