TÄHT7039: Radio astronomy and interferometry

Kaj Wiik & Silja Pohjolainen

Tuorla Observatory

Spring 2015

Some of the figures are from Wilson, Rohlfs, Hüttemeister: 'Tools of Radio astronomy'

Outline

Recap from lecture 5

Introduction to polarization

(Quasi)optical polarizers

Rotation measure

Radiometer sensitivity in presence of gain variations

The minimum detectable noise temperature of Dicke radiometer:

$$\Delta T_{min} = 2T_{sys}\sqrt{\frac{1}{\Delta\nu\tau} + \left(\frac{T_{sky} - T_{ref}}{T_{sys}} \cdot \frac{\Delta G}{G}\right)^2}, \qquad (1)$$

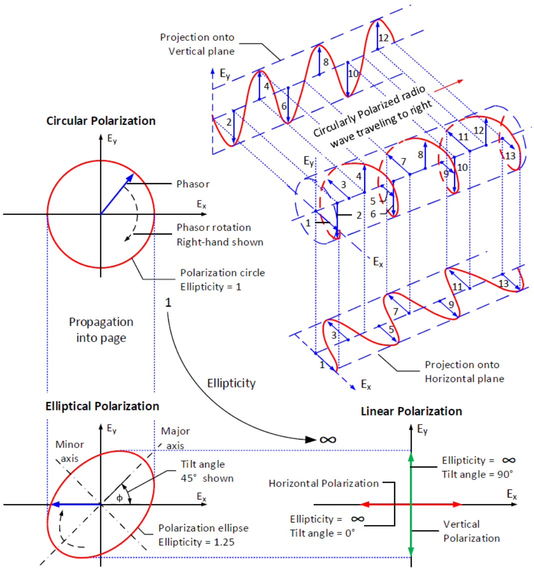
Gain variations are canceled totally if

- the reference and antenna noise temperatures are equal and
- the switching frequency is sufficiently high so that the gain can be considered nearly constant in the sky-reference cycle.

Pseudocorrelation radiometer

- It does not have $\sqrt{2}$ penalty in sensitivity because it observes the source continuously.
- It observes also the reference continuously, even the fastest variations get canceled.
- Like all switched radiometers, it must be *balanced*.

Introduction to polarization



Credit: Whitham D. Reeve

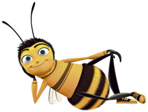
POLARISATION I:
FOR THE LOVE OF STOKES

Simon Ellingsen, CASS Radio School 2011
Thanks to Jimi Green

Outline

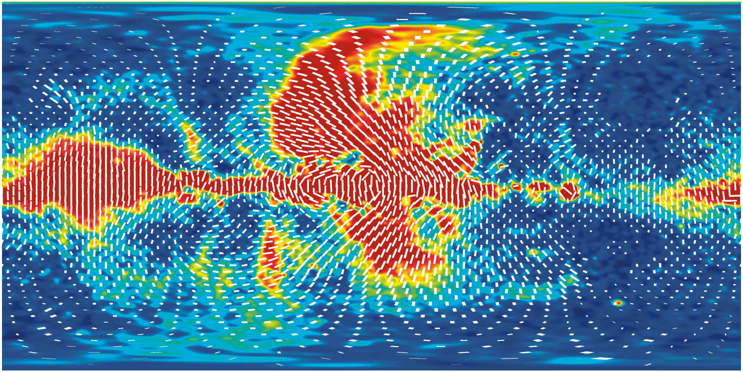
- Why study Polarimetry and what is Polarisation?
- Poincare and his spheres
- Jones and his vectors (and matrices)
- Stokes and his parameters
- How do we measure Polarisation?
- Leakages and Mueller’s Matrix
- Polarised beam effects
- Science with Polarisation:
  - Masers and Zeeman splitting

Why Polarimetry?

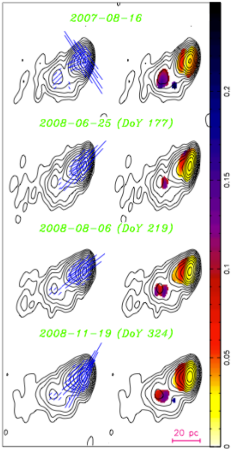


- Polarisation is fundamentally important to understanding the Universe
  - Provides insight into magnetic fields
- In optical astronomy, it's difficult to make polarimetric observations; in radio astronomy, they can be made easily, so why not use this to our advantage!
- (It's also very important to the Birds & the Bees, navigationally speaking, c.f. Rossel & Wehner,1984)

Why Polarimetry?



Why Polarimetry?

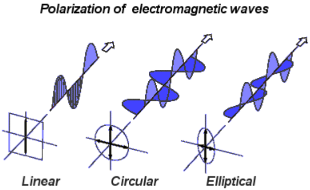


- 15 GHz VLBA polarization observations of PKS 1502+106 (Abdo et al. 2010).
- E-field vectors shown on left, fractional linear polarization shown on right, contours are total intensity.

What is Polarimetry & Polarisation?

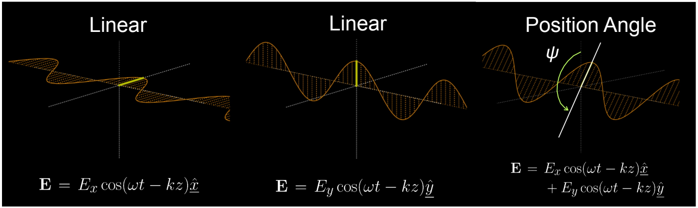
- Polarisation is the behaviour of the electric field with time.
- To simplify things we will start by considering monochromatic radiation.
- Astrophysical processes like synchrotron radiation can emit partially polarised emission, but *never* fully polarised.
- Interstellar matter can polarise random background emission or de-polarise polarised background emission.

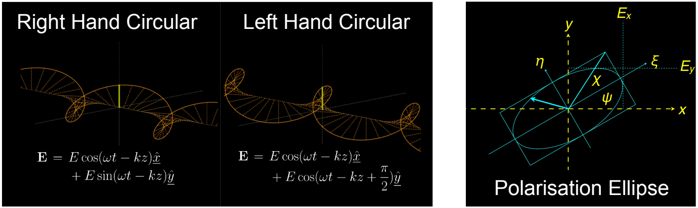
What is Polarisation?



- Linear: orthogonal components in phase with constant ratio of strengths giving constant direction of electric vector.
- Circular: orthogonal components 90° out of phase with equal amplitudes – electric vector traces circle.

What is Polarisation?





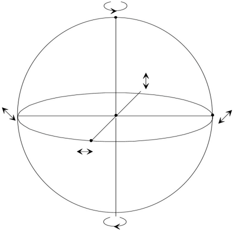
What is Polarisation?

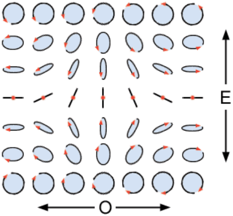
- Linearly polarised wave can be decomposed into two opposite handed circular waves.
- Conversely a circularly polarized wave can be decomposed into two orthogonal linear waves.
- Sum of two circular waves of unequal amplitude is elliptical.
- Sum of two orthogonal linears with a phase difference of between 0 and $\pi/2$ is also elliptical.

Jules Henri Poincaré

..and his sphere

- the spherical surface occupied by completely polarised states in the space of the vector
- Poles represent circular polarisations
  - Upper-hemisphere LHCP
  - Lower-hemisphere RHCP
- Equator represents linear polarisations with longitude representing tilt angle
- Latitude represents axial ratio





Robert Clarke Jones

..and his vectors

- Jones calculus is a matrix-based means of relating observed to incident fields.
- Vectors describe incident radiation and matrices the response of the instrument.
- The Jones Vector: $\begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix}$
- Examples:
  - Linearly (x-direction) polarised wave: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
  - Left-Hand Circularly polarised wave: $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$

Robert Clarke Jones

..and his matrices

- Effect of instrument described by 2x2 matrix:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}_0 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}_i$$

- Simple Examples:
  - Linear polariser: $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
  - Left-Hand Circular polariser: $\frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$
- In practice matrix elements complex.
- Important: Only applicable to ***completely*** polarised waves.

Sir George Gabriel Stokes
..and his parameters

- Defined by George in 1852
- Adopted for astronomy by Chandrasehkar in 1947.
- Can be used for partially polarised radiation.
- Not a vector quantity! Deals with power instead of electric field amplitudes.
- The correlator can produce ALL Stokes parameters simultaneously (not so easy in optical astronomy!)

Stokes Parameters

- I – total intensity and sum of any two orthogonal polarisations
- Q and U – completely specify linear polarisation
- V – completely specifies circular polarisation

$$I = E_{0x}^2 + E_{0y}^2$$
$$Q = E_{0x}^2 - E_{0y}^2$$
$$U = 2E_{0x}E_{0y}\cos\delta$$
$$V = 2E_{0x}E_{0y}\sin\delta$$

$$I = \langle E_x E_x^* \rangle + \langle E_y E_y^* \rangle$$
$$Q = \langle E_x E_x^* \rangle - \langle E_y E_y^* \rangle$$
$$U = \langle E_x E_y^* \rangle + \langle E_x^* E_y \rangle$$
$$V = i\left(\langle E_x E_y^* \rangle - \langle E_x^* E_y \rangle\right)$$

(For linear feeds)

Fractional Polarisations

- The total linearly polarised intensity is defined as:

$$P = \sqrt{U^2 + Q^2}$$

[for native linear feed]

- A linearly polarised source will have an intrinsic position angle on the sky that is given by:

$$\Theta = \frac{1}{2}\tan^{-1}\left(\frac{U}{Q}\right)$$

[for native linear feed]

- The circular polarisation will be just Stokes V.
- Stokes parameters often presented as percentages of the total intensity.
- Since radio sources are *never* fully polarised, then the fractional linear and circular polarisation will always be <1

How do we measure it?

- Stokes parameters are the auto-correlation & cross-correlation products returned from the correlator, but input to the correlator can come from different feed types.
- Feeds normally designed to approximate pure linear or circular (known as 'native linear' or 'native circular')
  - Linear Feeds – intrinsically accurate & provide true linear response.
  - Circular Feeds – less accurate & frequency dependent response.

How do we measure it?

- Output of native linear feed is E_x and E_y field voltages, so:
  - I from XX+YY
  - Q from XX-YY
- Native circular adds 90° phase to X, so:
  - I from XX+YY (or LL+RR)
  - V from XX-YY (or LL-RR)

Stokes Parameters

- For circular feeds Q and V swap round..

Linear	Circular
$XX = I + Q$	$RR = I + V$
$YY = I - Q$	$LL = I - V$
$XY = U + iV$	$RL = Q + iU$
$YX = U - iV$	$LR = Q - iU$

But is it really that simple?

- Do we just plug in our computer and get {I,Q,U,V} out of the correlator?
- No, there are leakages!
  - The total intensity can leak into the polarised components (I into {Q,U,V}).
  - The linear polarisation can leak into the circular ({Q,U} into V).
  - … and all combinations and permutations are allowed!
- Without correcting for leakage, you're not going to get proper Stokes parameters!

Hans Mueller

..and his matrix

- The leakage of each polarisation into the other can be measured and quantified in a 4x4 matrix first proposed by Mueller in 1943.

$$M = \begin{bmatrix} m_{II} & m_{IQ} & m_{IU} & m_{IV} \\ m_{QI} & m_{QQ} & m_{QU} & m_{QV} \\ m_{UI} & m_{UQ} & m_{UU} & m_{ii} \\ m_{VI} & m_{VQ} & m_{VU} & m_{VV} \end{bmatrix}$$

The Mueller Matrix

Correlator Output   Incoming Radiation

$$\begin{bmatrix} XX + YY \\ XX - YY \\ XY \\ YX \end{bmatrix} = M \bullet \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

Example (simple) Mueller Matrices

- If feeds were perfect:
  - Dual linear feed: M is unitary
  - Dual linear feed rotated 45°: Q and U interchange and sign change for rotation:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - A dual linear feed rotated 90°: signs of Q and U reversed:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- As Alt-Az telescope tracks source, feed rotates on sky by the parallactic angle (PA):

$$M_{sky} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2PA & \sin 2PA & 0 \\ 0 & -\sin 2PA & \cos 2PA & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The more general Mueller Matrix

- For a (realistic) dual linear feed:

$$M = \begin{bmatrix} 1 & \left(-2\varepsilon\sin\phi\sin 2\alpha + \frac{\Delta G}{2}\cos 2\alpha\right) & 2\varepsilon\cos\phi & \left(2\varepsilon\sin\phi\cos 2\alpha + \frac{\Delta G}{2}\sin 2\alpha\right) \\ \frac{\Delta G}{2} & \cos 2\alpha & 0 & \sin 2\alpha \\ 2\varepsilon\cos(\phi+\varphi) & \sin 2\alpha\sin\varphi & \cos\varphi & -\cos 2\alpha\sin\varphi \\ 2\varepsilon\sin(\phi+\varphi) & -\sin 2\alpha\cos\varphi & \sin\varphi & \cos 2\alpha\cos\varphi \end{bmatrix}$$

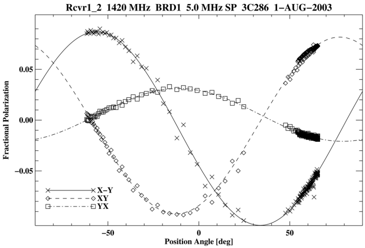
- The Mueller matrix has 16 elements, but ONLY 7 INDEPENDENT PARAMETERS. The matrix elements are not all independent.

Calculating the Mueller Matrix

- For a perfect system, as we track a polarised source across the sky the parallactic angle changes and this should produce:
  - For XX-YY: cos2(PA_{az}+PA_{src}), centred at zero.
  - For XY: sin 2(PA_{az}+PA_{src}), centred at zero
  - For YX: zero (most sources have zero circular polarisation)

Calculating the Mueller Matrix

- But, what we find is:



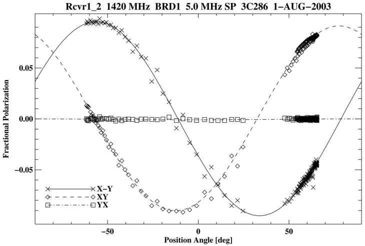
DELTAG = –0.017 ± 0.006
PSI = –16.7 ± 1.8
ALPHA = +0.4 ± 0.9
EPSILON = +0.004 ± 0.001
PHI = +152.3 ± 23.0
Q_{SRC} = –0.037 ± 0.002
U_{SRC} = –0.085 ± 0.002
POL_{SRC} = +0.093 ± 0.002
PA_{SRC} (UNCORRECTED FOR M_{ASTRO}**) = –56.81 ± 0.64**
NR GOOD POINTS: X–Y = 149 XY = 158 YX = 160 / 160

Mueller Matrix:

1.0000	–0.0083	–0.0065	0.0033
–0.0083	0.9999	0.0001	0.0145
–0.0053	–0.0042	0.9581	0.2865
0.0052	–0.0139	–0.2865	0.9580

Calculating the Mueller Matrix

- Which enables the matrix to be calculated and the observations corrected to give what we expect:



DELTAG = 0.000 ± 0.006
PSI = +0.3 ± 1.8
ALPHA = −0.0 ± 0.9
EPSILON = +0.000 ± 0.001
PHI = −39.8 ± 235.4
Q_{SRC} = −0.037 ± 0.002
U_{SRC} = −0.085 ± 0.002
POL_{SRC} = +0.093 ± 0.002
PA_{SRC} (**UNCORRECTED FOR M_{ASTRO}**) = −56.82 ± 0.64
NR GOOD POINTS: X–Y = 149 XY = 158 YX = 158 / 160

Mueller Matrix:

1.0000	0.0002	0.0006	−0.0005
0.0002	1.0000	0.0000	−0.0012
0.0006	−0.0000	1.0000	−0.0044
−0.0005	0.0012	0.0044	1.0000

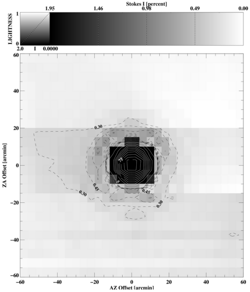
Putting it all together

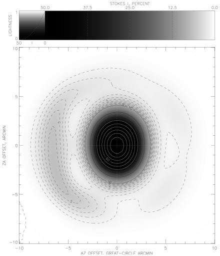
- In the end what we are trying to do is relate products from our correlator to the intrinsic polarized radiation from the source.
- So we need to correct the raw correlator outputs for
  - Imperfections in the receiver (leakages).
  - The orientation of the receiver with respect to the telescope structure (maybe).
  - The changing parallactic angle (maybe).
  - Any measured propagation related polarization effects (e.g. Faraday rotation).

Beam Effects

- For point sources, all of the previous is fine.
- What if the source you're looking at is extended compared to the telescope beam?
- There are instrumental beam effects that can confuse the measurement of extended polarised signals. They are…
  - Squint
  - Squash

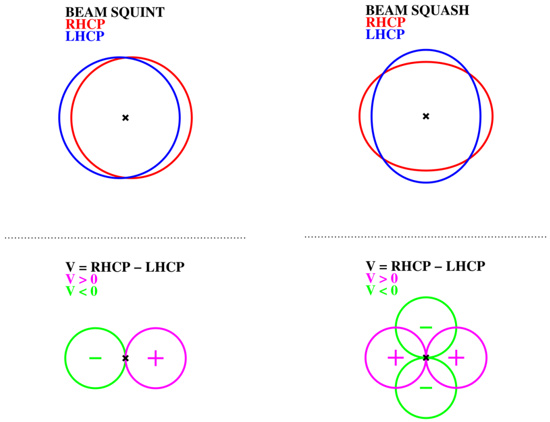
Stokes I response



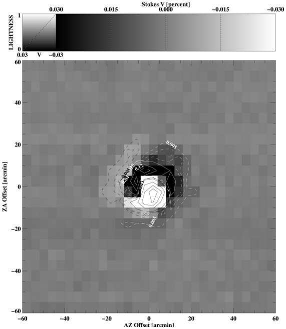


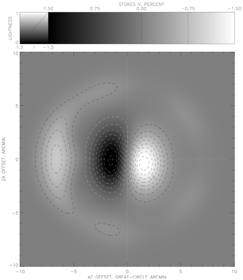
Heiles et al. 2001

Beam Squint & Squash



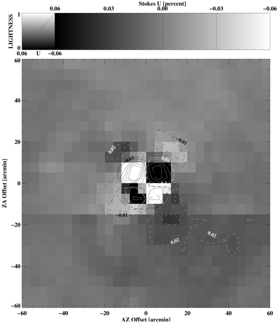
Squint in action

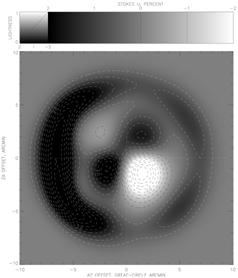




Heiles et al. 2001

Squash in action





Heiles et al. 2001

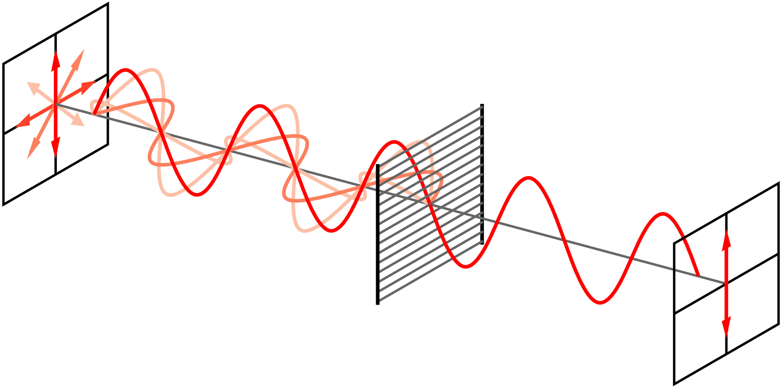
Summary

- Polarisation in radio astronomy very important to improving our knowledge & understanding.
- Can describe polarisation with the Polarisation Ellipse and the Poincare Sphere.
- Dr. Jones offers a vector representation for ideal cases of completely polarised emission.
- Mueller and his matrices are the best option for real situations.
- There are Linear and Circular feed types, must account for which you are using.
- **Understanding the polarisation properties of your dish is fundamental to successful observations!**
- (Masers offer exciting science opportunities!)

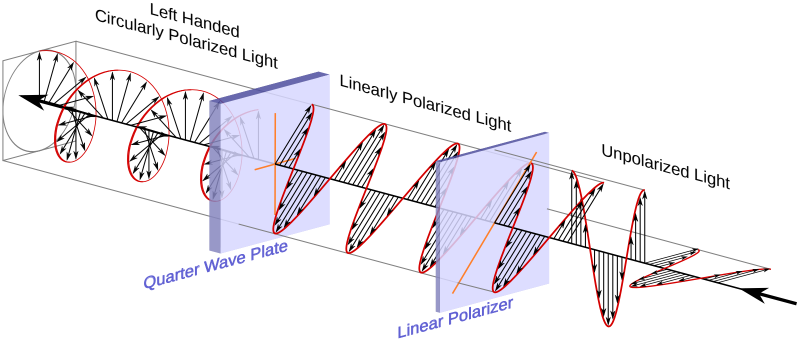
Useful References

- Heiles, C. 'A Heuristic Introduction to Radioastronomical Polarisation' (2002) ASP 278
- Tinbergen, J. 'Astronomical Polarimetry' (1996), Cambridge University Press (Cambridge, UK)
- Stutzman, W. 'Polarisation in Electromagnetic Systems' (1993), Artech House (Norwood, MA, USA)
- Radhakrishnan. Polarisation. URSI proceedings (1990) pp. 34
- Hamaker et al. Understanding radio polarimetry. I. Mathematical foundations. Astronomy and Astrophysics Supplement (1996) vol. 117 pp. 137
- Born and Wolf: 'Principle of Optics', Chapters 1 and 10

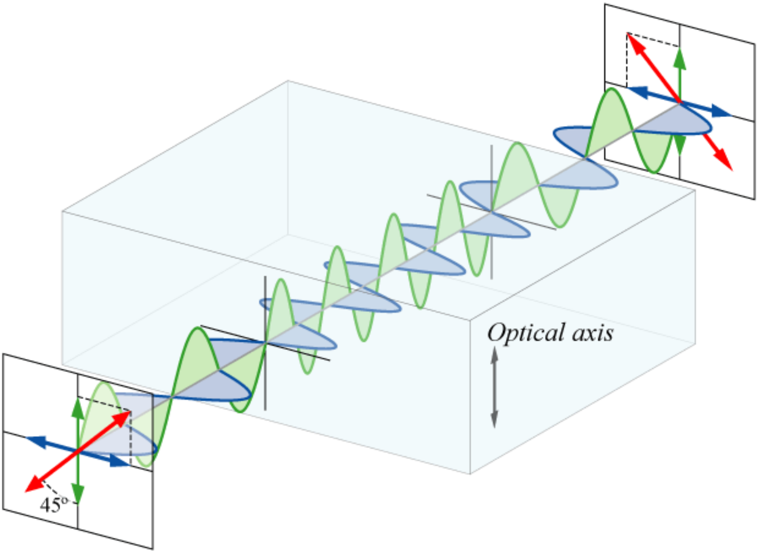
Wire grid polarizer (linear)



Circular polarizer ($\lambda/4$ plate)



$\lambda/2$ plate



Faraday polarization rotation and rotation measure

Polarimetric observations give information of the magnetic field strength and direction and also of electron (charge) density.
This is due to Faraday effect where the electric vector polarization angle (EVPA) of a linearly polarized wave rotates when it travels trough a medium with plasma and magnetic field:

$$\chi^{\mathrm{lin}}(\lambda) = \chi_0^{\mathrm{lin}} + \mathrm{RM} \cdot \lambda^2, \tag{2}$$

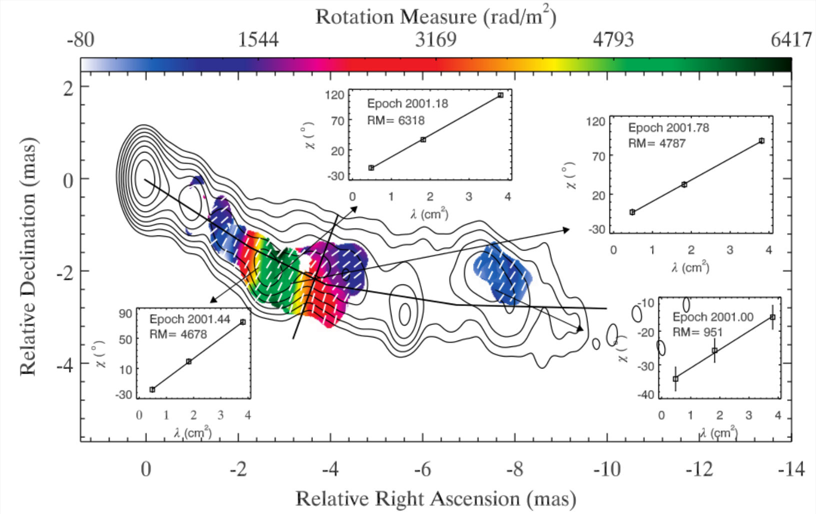
where

$$\mathrm{RM} = 0.81 \int_{\mathrm{source}}^{\mathrm{observer}} n_e \overrightarrow{B} \cdot \mathrm{d}\overrightarrow{l}. \tag{3}$$

χ_0^{lin} is the original EVPA, RM is *rotation measure*, n_e is electron density (cm^{-3}, B is magnetic field strength (μG), and l is travelled distance in parsec.

(Rohlfs et al. book uses χ in different context, because of this χ^{lin} is used here for EVPA)

Rotation measure in 3C 120



Gómez et al. ApJ 681: L69–L72

That's it!