TÄHT7039: Radio astronomy and interferometry

Kaj Wiik & Silja Pohjolainen

Tuorla Observatory

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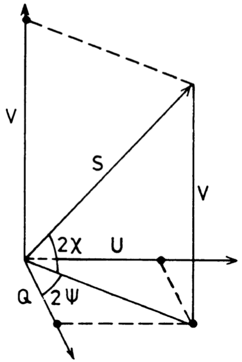
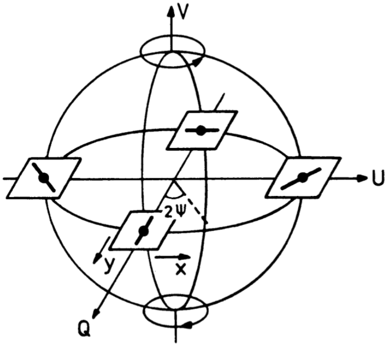
Based partly on 'Essential radio astronomy' from http://www.cv.nrao.edu/course/astr534/Interferometers1.html by J. J. Condon and S. M. Ransom and Ivan Marti-Vidals (OSO, ALMA Nordic ARC) tutorial.

Outline

Recap: Poincaré sphere and Stokes parameters





If 2Ψ is interpreted as longitude and 2χ as latitude on a sphere with a radius of S_0, all polarization states can be expressed on a surface of a sphere.

The positions on equator correspond to linear polarization, north pole right hand circular and south pole left hand circular polarization. The axes correspond to *Stokes parameters*.

Recap: Stokes parameters II

The previous definitions can be used to express the polarization state of a wave exactly but they do not correspond directly to observable quantities. By expressing the parameters using field vectors and their phase shift, we get:

$$
\begin{aligned}
S_0 &= I = E_1^2 + E_2^2 \\
S_1 &= Q = E_1^2 - E_2^2 \\
S_2 &= U = 2E_1E_2\cos\delta \\
S_3 &= V = 2E_1E_2\sin\delta
\end{aligned}
\tag{1}
$$

Special cases:

- Right-handed circular polarization: $I = V = S$ and $Q = U = 0$
- Left-handed circular polarization: $I = S,\ V = -S$ and $Q = U = 0$
- Linear polarization: $I = S,\ V = 0,\ Q = I\cos 2\Psi$ and $U = I\sin 2\Psi$

Recap: Faraday polarization rotation and rotation measure

Polarimetric observations give information of the magnetic field strength and direction and also of electron (charge) density.
This is due to Faraday effect where the electric vector polarization angle (EVPA) of a linearly polarized wave rotates when it travels trough a medium with plasma and magnetic field:

$$\chi^{\mathrm{lin}}(\lambda) = \chi_0^{\mathrm{lin}} + \mathrm{RM} \cdot \lambda^2, \tag{2}$$

where

$$\mathrm{RM} = 0.81 \int_{\mathrm{source}}^{\mathrm{observer}} n_e \overrightarrow{B} \cdot \mathrm{d} \overrightarrow{l}. \tag{3}$$

χ_0^{lin} is the original EVPA, RM is *rotation measure*, n_e is electron density (cm^{-3}, B is magnetic field strength (μG), and l is travelled distance in parsec.

(Rohlfs et al. book uses χ in different context, because of this χ^{lin} is used here for EVPA)

Interferometry

THE QUEST FOR RESOLUTION

Resolution = Observing wavelength / Telescope diameter

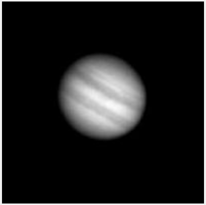
Angular	Optical (5000A)		Radio (4cm)	
Resolution	**Diameter**	**Instrument**	**Diameter**	**Instrument**
1′	**2mm**	**Eye**	**140m**	**GBT+**
1″	**10cm**	**Amateur Telescope**	**8km**	**VLA-B**
0.″05	**2m**	**HST**	**160km**	**MERLIN**
0.″001	**100m**	**Interferometer**	**8200km**	**VLBI**

Atmosphere gives 1" limit without corrections which are easiest in radio

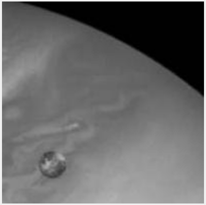
Jupiter and Io as seen from Earth

1 arcmin

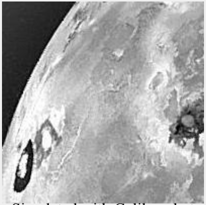
1 arcsec



0.05 arcsec



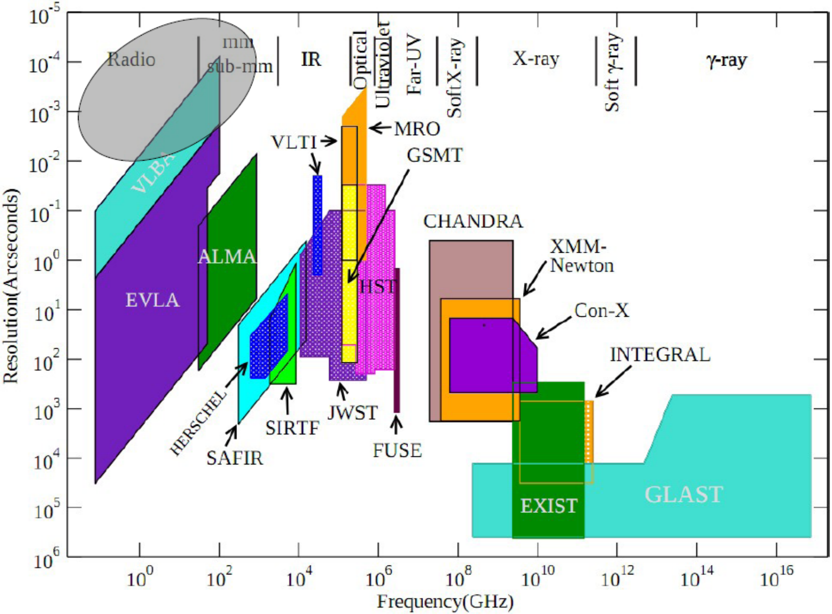
0.001 arcsec



Simulated with Galileo photo

From Walker (2002)

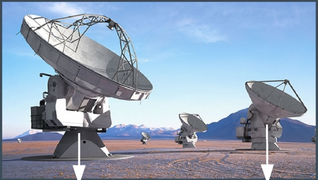
EM frequency-resolution space



Interferometers

- The interference is computed as the *signal cross-correlation*.

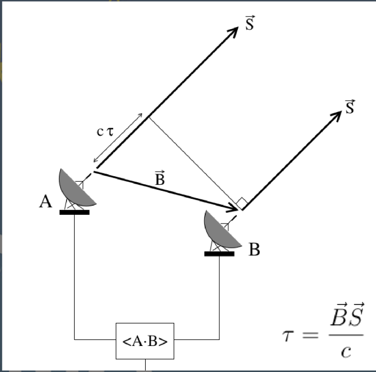
- A visibility is the coherent time average of this correlation.



$s_1(t_i)$ $\quad$ $s_2(t_i)$

$S_1(\nu_k)$ $\quad$ $S_2(\nu_k)$

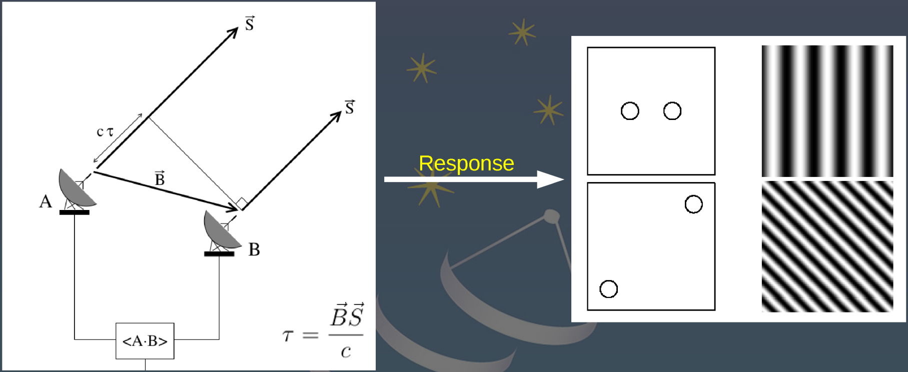
$V_k = < S_1(\nu_k)\, S_2(\nu_k)^* >$



$$Re = G_A G_B \frac{I}{2} \cos \omega\tau$$

$$Im = G_A G_B \frac{I}{2} \sin \omega\tau$$

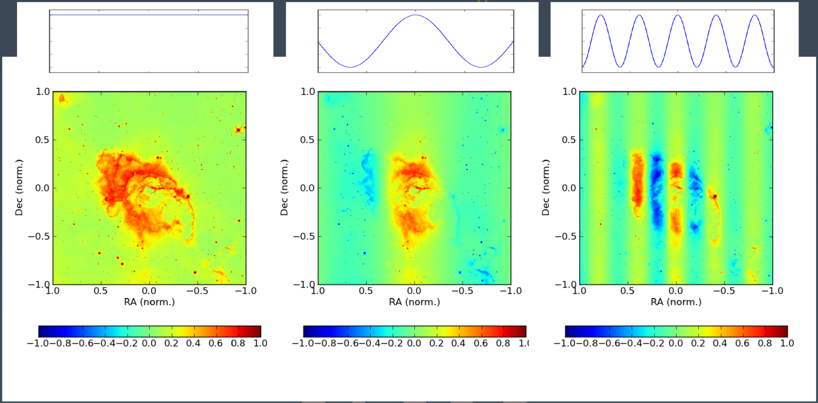
Interferometers



If the source observed is not point-like, the visibility is the integrated response (i.e., the integral of the product of the source intensity distribution by the response “sky fringes”).

BASICS OF INTERFEROMETRY

FOURIER TRANSFORM !



-IF THE FRINGE IS WIDE (*LOW* SPATIAL FREQUENCY), THE INTEGRAL IS SENSITIVE TO *LARGE* STRUCTURES.

-IF THE FRINGE IS NARROW (*HIGH* SPATIAL FREQUENCY), THE INTEGRAL CANCELS FOR LARGE STRUCTURES AND IS SENSITIVE TO *SMALL* STRUCTURES.

BASICS OF INTERFEROMETRY

FOURIER TRANSFORM !

$$V(u) = \mathcal{F}(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\,(\cos(2\pi\, u\, x) - i \sin(2\pi\, u\, x)))$$

Function to transform

Spatial frequency

Complex “fringes”

- THE DOMAIN OF THE FOURIER TRANSFORM ARE THE *SPATIAL FREQUENCIES*.

- EACH VALUE OF THE FOURIER TRANSFORM OF A FUNCTION IS THE INTEGRAL OF THE PRODUCT OF THE FUNCTION BY A (COMPLEX) “FRINGE” OF A GIVEN SPATIAL FREQUENCY.

Aperture Synthesis

The value of a visibility is related to the source structure (i.e., the intensity distribution) and the relative position of the telescopes.

$I(x,y)$ → Source structure at a given spectral channel. The image (i.e., sky) coordinates are (x,y).

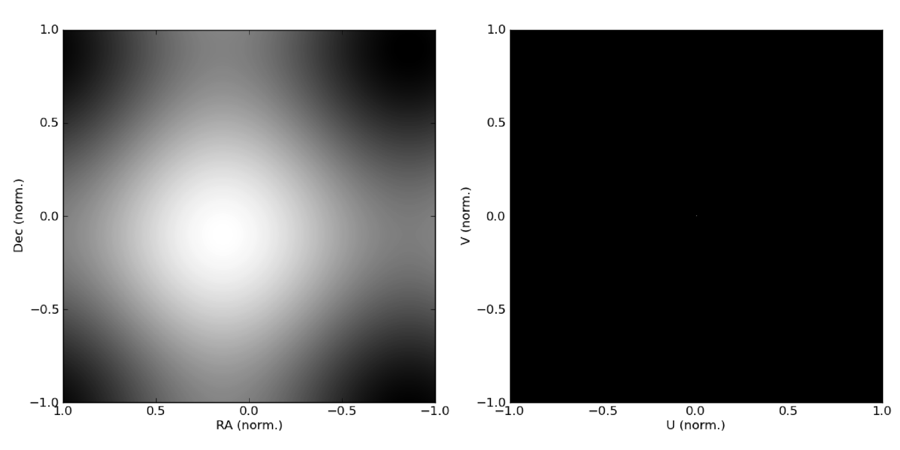
u, v → Coordinates of the *baseline* vector (i.e., the position of one telescope relative to the other) projected in the plane orthogonal to the source position.

V λ → Visibility and observing wavelength (both at the given spectral channel).

THEN (for compact sources and/or small fields of view):

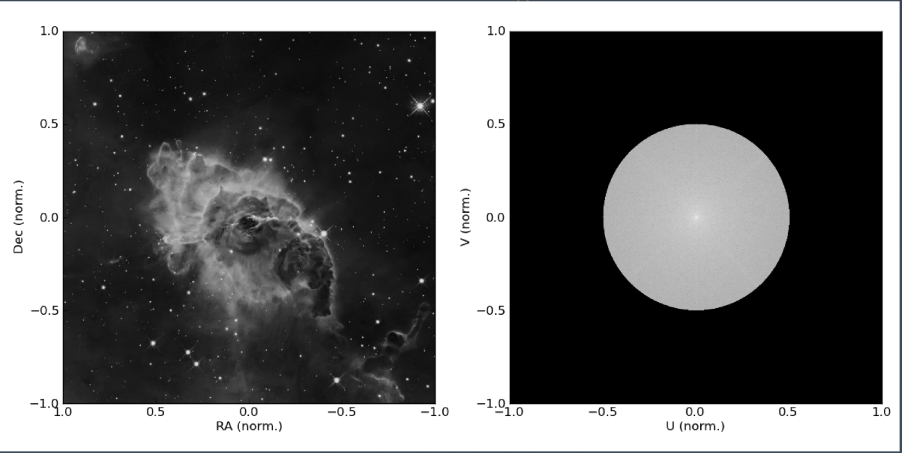
$$V\left(\frac{u}{\lambda}, \frac{v}{\lambda}\right) = \mathcal{F}\left[I(x,y)\right]$$

BASICS OF INTERFEROMETRY



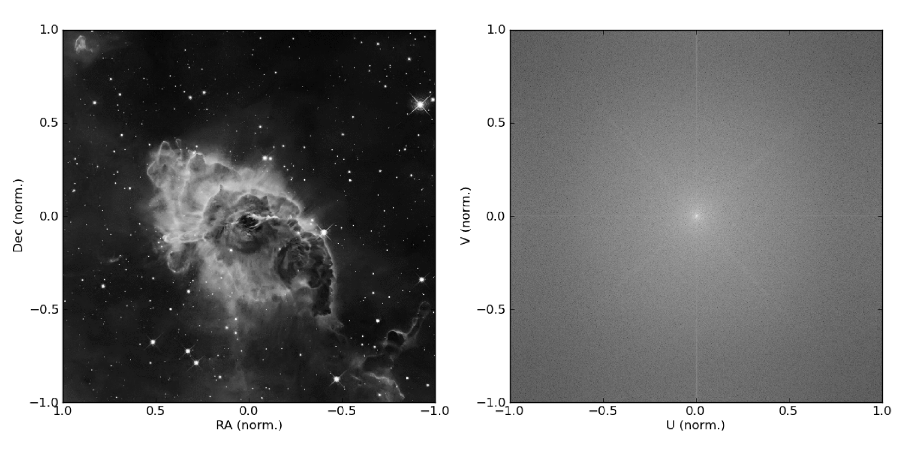
FOURIER TRANSFORM?

BASICS OF INTERFEROMETRY



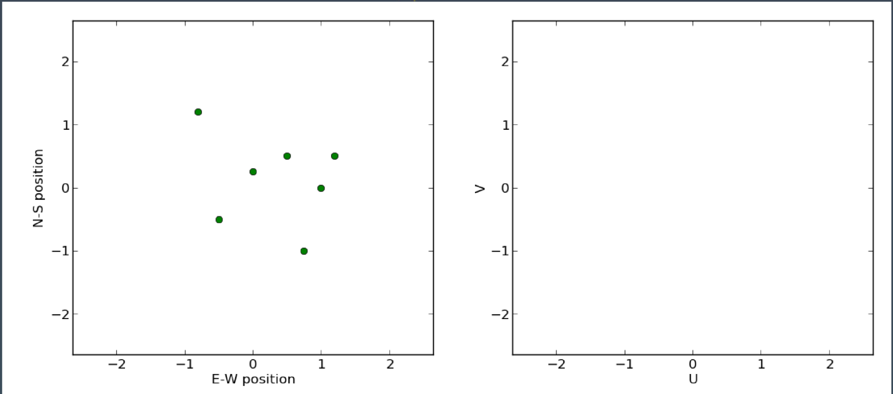
FOURIER TRANSFORM?

BASICS OF INTERFEROMETRY



FOURIER TRANSFORM?

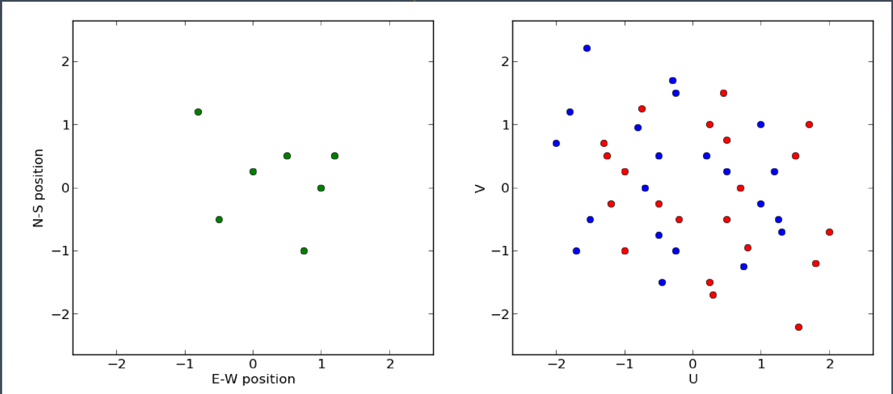
Aperture Synthesis



Monochromatic observations

$$V\left(\frac{u}{\lambda},\frac{v}{\lambda}\right)=\mathcal{F}\left[I\left(x,y\right)\right]$$

Aperture Synthesis



Monochromatic observations

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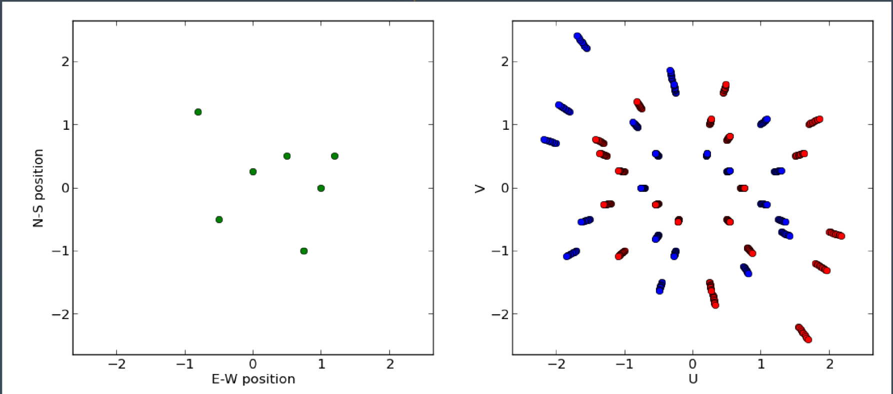


Bandwidth of 10%

$$V\left(\frac{u}{\lambda},\frac{v}{\lambda}\right)=\mathcal{F}\left[I\left(x,y\right)\right]$$

But reality is not that beautiful!

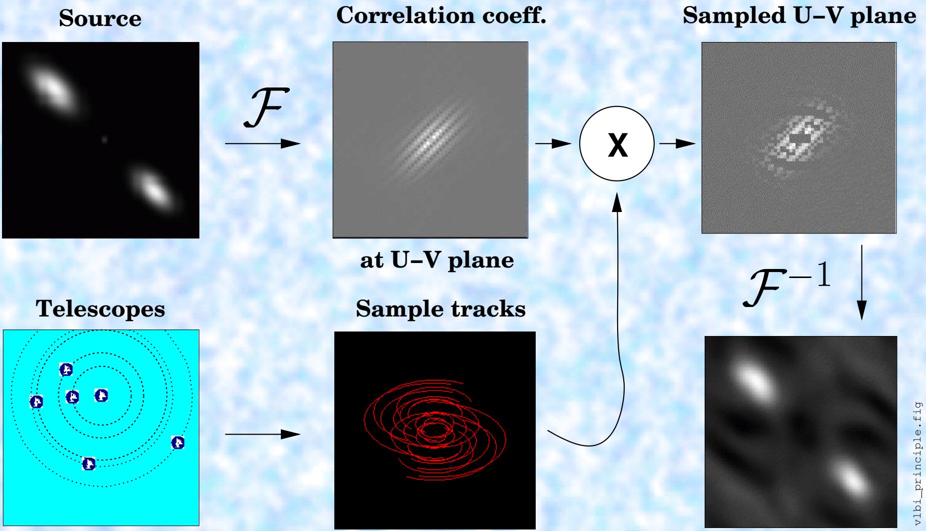
The measured visibilities are corrupted in many different ways.

- ATMOSPHERE:
  - Opacity (amplitude bias)
  - Water vapor (phase instabilities)

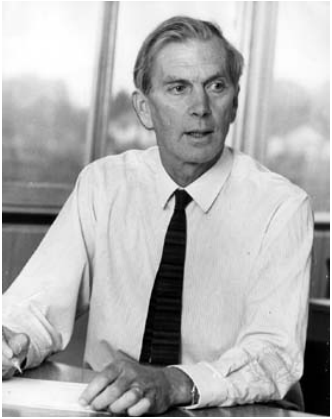
- INSTRUMENT:
  - Gain curve and pointing (varies with elevation and time)
  - Parallactic angle and leakage (affect polarization)
  - Bandpass (affects source spectrum)

- CORRELATOR & OTHERS:
  - Antenna positions (aberrating baseline-dependent effects).
  - Bandpass of digital filters (affects FDM ampl. calibration).
  - RFI.

VLBI (Very Long Baseline Interferometry)



How it all begun (here and there)



Sir Martin Ryle (1918 - 1984)

Jorma J. Riihimaa (1933 - 2011)

K.39. Teekkari J.J. Riihimaa etuvahvistinta virittämässä.

K. 36. "Jätepuuantenni". Toinen kahdesta neljän kokoaaltodipolin ryhmästä Viikin koetilan alueella. Taustalla "työnjohtajan koppi". (Heinäkuu 1953)

A new radio interferometer and its application to the observation of weak radio stars

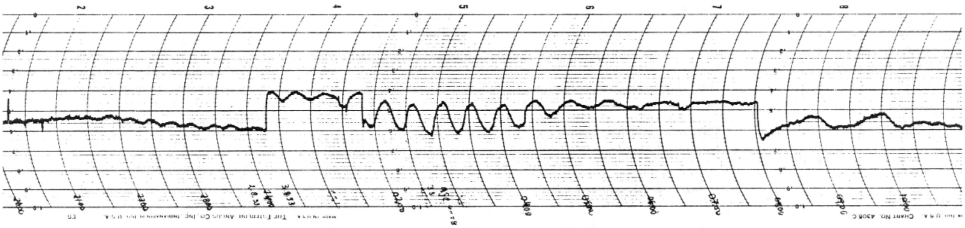
By M. Ryle

(*Communicated by Sir Lawrence Bragg, F.R.S.—Received* 19 *June* 1951—*Revised* 10 *October* 1951)

A new type of radio interferometer has been developed which has a number of important advantages over earlier systems. Its use enables the radiation from a weak 'point' source such as a radio star to be recorded independently of the radiation of much greater intensity from an extended source. It is therefore possible to use a very much greater recorder sensitivity than with earlier methods. It is, in addition, possible to use pre-amplifiers at the aerials, and the resolving power which may be used is therefore not restricted by attenuation in the aerial cables.

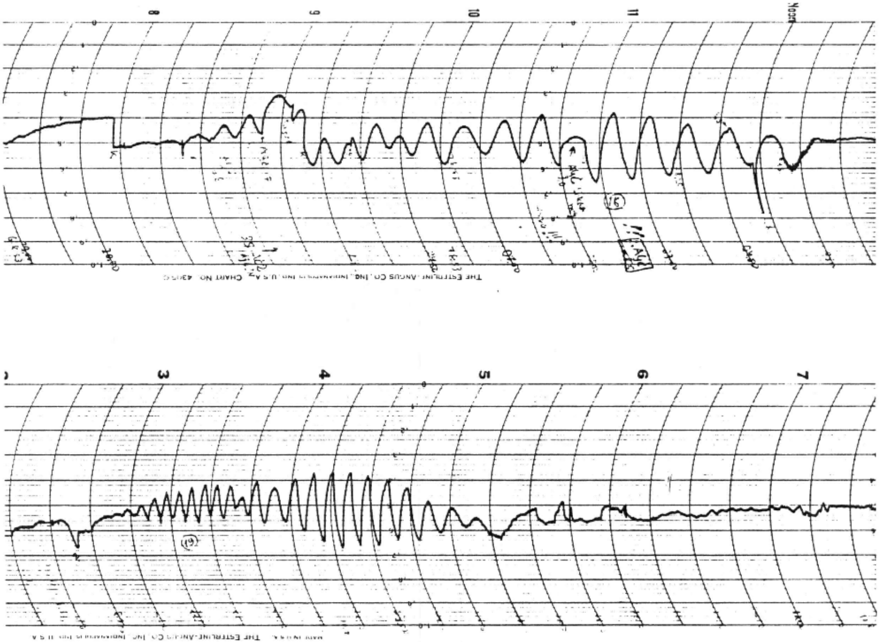
Besides improved sensitivity, the new system has a number of other advantages, particularly for the accurate determination of the position of a radio source. Unlike earlier systems the accuracy of position finding is not seriously affected by rapid variations in the intensity of the radiation. It also has important applications to the measurement of the angular diameter and polarization of a weak source of radiation.

The new system has been used on wave-lengths of 1·4, 3·7, 6·7 and 8 m for the detection and accurate location of radio stars, and for the investigation of the scintillation of radio stars. It has also been used in a number of special experiments on the radiation from the sun. The results which have been obtained in these experiments have confirmed the advantages predicted analytically.



1953

K.43. Ensimmäinen rekisteröinti Cass A:n ohikulusta
2-3.8.1953. (81.5 MHz)



K.44. Kojeiston parantelua. Oikealla näkyvät Cyg A:n ja Cass A:n perättäiset ohikulut (pienemmällä rek.pap. nopeudella).

Interferometry in Finland

- First fringes by Riihimaa (OH2PX,OH8PX) in 1953
- 1973 Riihimaa started decametric observations with a two-LPDA interferometer in Oulu
- 1979 First tests in Mets"ahovi with the 13.7m telescope at 5 GHz
- 1991 First observations at 22 GHz with Finnish-built equipment
- 1995 First mm-VLBI at 86 GHz
- 2001 First 2mm (150 GHz) observations
- Fast development of eVLBI ({www.metsahovi.fi})

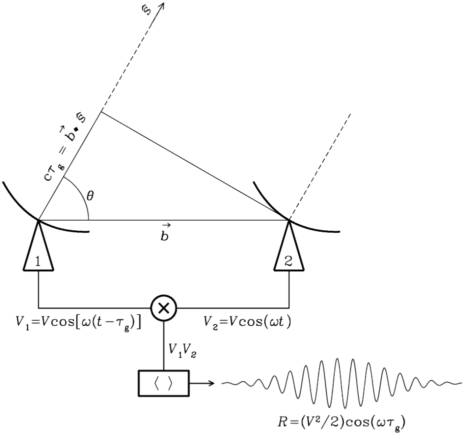
e- VLBI:

connecting remote telescopes in real- time

Huib Jan van Langevelde
Joint Institute for VLBI in Europe
Sterrewacht Leiden

Two-element monochromatic interferometer

Interferometry is based on correlation (multiplication and averaging) of signals from two radio telescopes.

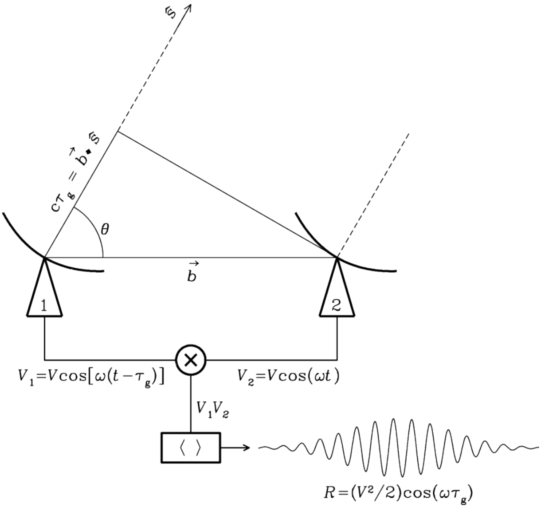


Instead of focusing the electromagnetic wave using lenses and mirrors we *sample the aperture field* and convert the samples to an image mathematically.

Baselines and visibilities

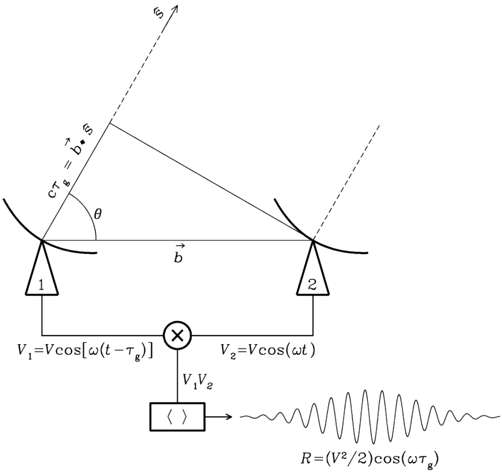
- One pair of telescopes gives one *baseline* and one *visibility* in the *uv-plane*.
- Correlating signals from multiple telescopes pairwise gives visibilities from different baselines.
- N telescopes give $N(N-1)/2$ independent baselines and visibilities.

Geometry of a two-element interferometer



$\vec{b}$ = baseline vector, $\hat{s}$ = unit vector to source,
$\tau_g = \vec{b} \cdot \hat{s}/c$ = geometric delay between signals.

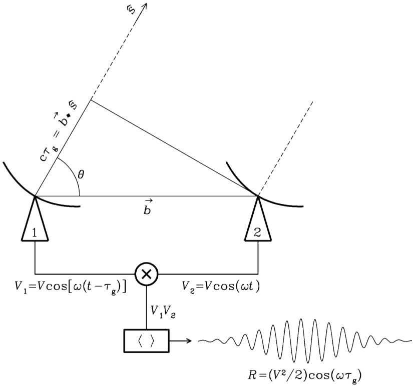
Signals from antennas



At frequency $\nu = \omega/(2\pi)$ the signals from antennas are

$$V_1 = V\cos[\omega(t-\tau_g)] \quad \text{and} \quad V_2 = V\cos(\omega t)\ . \tag{4}$$

Correlation

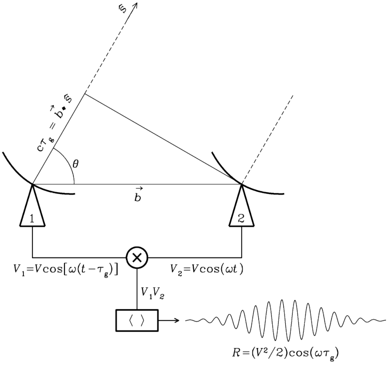


After multiplication the product is averaged with a timescale of typically seconds), so that the term $\cos(2\omega t - \omega\tau_{\rm g})$ averages out.

Multiplication result:

$$V_1 V_2 = V^2 \cos(\omega t) \cos[\omega(t - \tau_{\rm g})] = \left(\frac{V^2}{2}\right)[\cos(2\omega t - \omega\tau_{\rm g}) + \cos(\omega\tau_{\rm g})] \tag{5}$$

Averaged multiplication = correlation

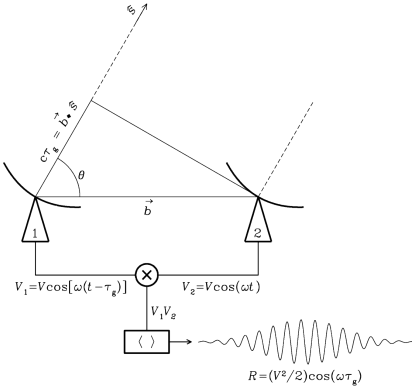


The averaged product
i.e. correlation is

$$R = \langle V_1 V_2 \rangle = \left(\frac{V^2}{2}\right)\cos(\omega\tau_{\mathrm{g}})\,. \tag{6}$$

Output amplitude $V^2/2$ is proportional to the point-source flux density multiplied by the geometric mean of the telescope effective areas.

Fringes



The correction result $\left(\frac{V^2}{2}\right)\cos(\omega\tau_g)$ varies sinusoidally because the source direction is slowly changing (earth rotation). These sinusoids are called *fringes*. *Fringe phase* is

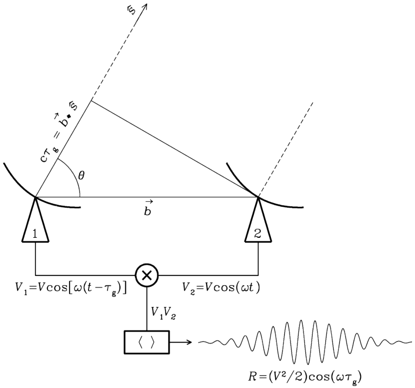
$$\phi = \omega\tau_g = \frac{\omega}{c} b\cos\theta \tag{7}$$

and depends on direction as follows:

$$\frac{d\phi}{d\theta} = \frac{\omega}{c} b\sin\theta = 2\pi\left(\frac{b\sin\theta}{\lambda}\right). \tag{8}$$

Fringes are tapered by the primary beam if telescopes are not tracking the source.

Fringe phase and source position

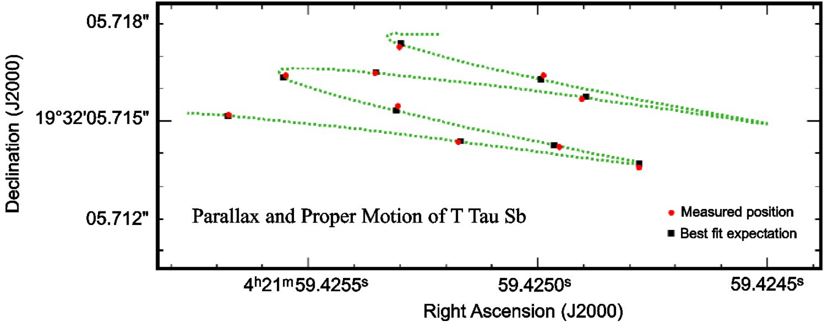


$$\frac{d\phi}{d\theta} = \frac{\omega}{c} b \sin\theta = 2\pi\left(\frac{b\sin\theta}{\lambda}\right) \tag{9}$$

One fringe period $\Delta\phi = 2\pi$ corresponds to an angular change of $\Delta\theta = \lambda/(b\sin\theta)$.
Because of this, it is very sensitive measure of source position if the projected baseline $b\sin\theta$ is very long in wavelengths.

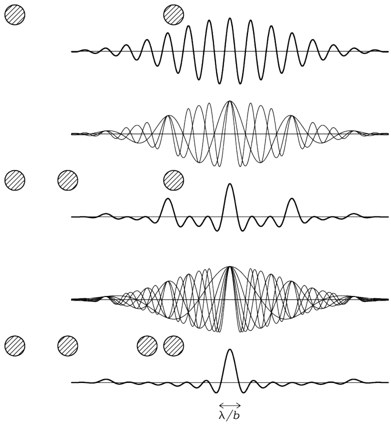
Fringe phase does not depend on telescope direction i.e. tracking errors but depends only on time. Interferomteres can determine positions with better accuracy than any other methods. Absolute positions in milliarcseconds and relative down to tens of microarcseconds.

Example: Parallax and proper motion of T Tau Sb



The VLBA was used to measure the parallax and proper motion of the radio star T Tau Sb. The measured parallax $\Pi = 6.82 \pm 0.03$ milli-arcsec implies a distance $D = 146.7 \pm 0.6$ pc (Loinard et al. 2007, ApJ, 671, 546), an improvement by nearly two orders of magnitude over the distance $D = 177^{+68}_{-39}$ pc obtained by the Hipparcos astrometry satellite.

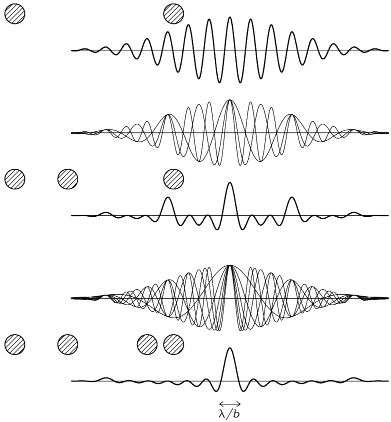
Fourier spatial frequencies



Isotropic antenna elements would fill the sky with a sinusoid response. Individual responses of (real world) directive antenna elements are multiplied with the sinusoid, result is a 'sinusoid pulse' pattern.

One baseline is sensitive to only one spatial frequency ($b \sin \theta / \lambda$). To improve response, more Fourier components are needed.

Fourier spatial frequencies and synthesized beam



An interferometer with N antennas contains $N(N-1)/2$ pairs of antennas i.e. baselines
These cases have

- one baseline b
- three baselines b, $b/3$, $2b/3$
- six baselines b, $b/6$, $2b/6$, $3b/6$, $4b/6$, $5b/6$

When the number of **unique** baselines are increased, the synthesized beam approaches Gaussian with angular resolution $\approx \lambda/b$.

That's it!