TÄHT7039: Radio astronomy and interferometry

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Based partly on 'Essential radio astronomy' from http://www.cv.nrao.edu/course/astr534/Interferometers2.html and http://www.cv.nrao.edu/course/astr534/Interferometers2.html by J. J. Condon and S. M. Ransom.

Outline

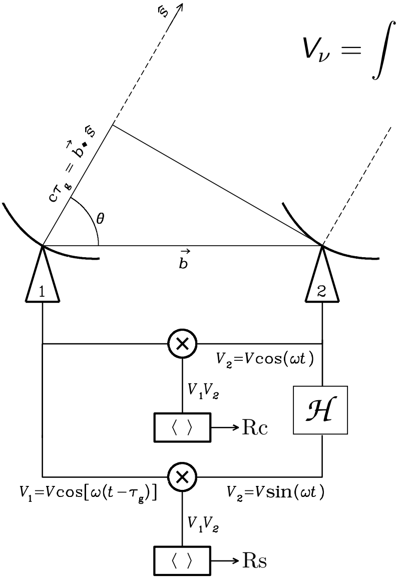
Recap from lecture 8

Recap

Interferometers in Three Dimensions

Amplitude calibration

Recap: complex correlator response



$$V_\nu = \int I_\nu(\hat{s}) \exp(-i2\pi \vec{b} \cdot \hat{s}/\lambda)\ \mathrm{d}\Omega$$

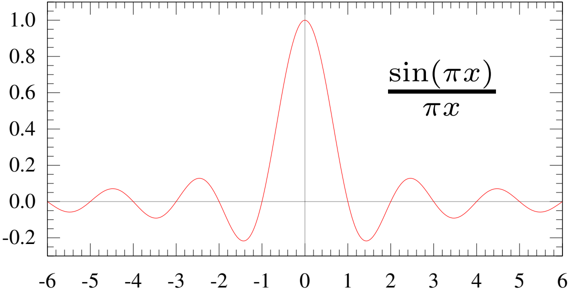
Complex correlator can be implemented simply by introducing 90° phase shift or a *Hilbert transform* $\mathcal{H}$ to the second branch of the correlator.

Complex visibility is

$$V \equiv R_\mathrm{c} - iR_\mathrm{s} = Ae^{-i\phi}$$

Recap: Effect of finite observing bandwidth

$$V = \int I_\nu(\hat{s}) \mathrm{sinc}(\Delta\nu\tau_\mathrm{g}) \exp(-i2\pi\nu_\mathrm{c}\tau_\mathrm{g}) \, \mathrm{d}\Omega \ .$$



To reduce the effect of bandwith smearing, the field of view times fractional bandwidth should be much less than the synthesized beam (in practise observe in frequency slices):

$$\frac{\Delta\theta\Delta\nu}{\nu} \ll \theta_\mathrm{s}$$

Recap: Time smearing

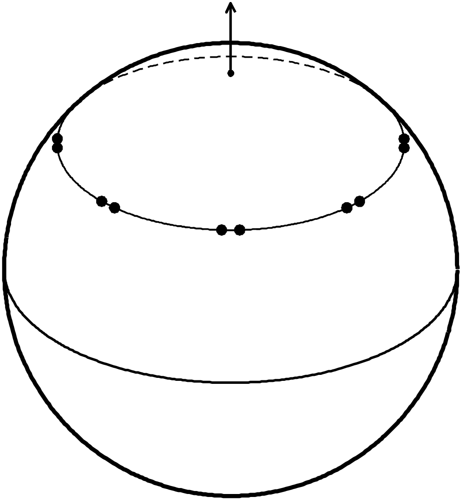
Like finite bandwith, finite correlator integration time smears images with large fields. This is because Earth's rotation moves the source position in the frame of the interferometer.

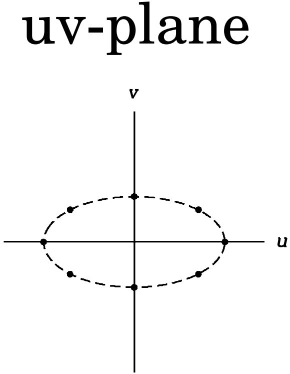
This should be kept much smaller than the synthesized beam $\theta_{\rm s} \approx \lambda/b$. E.g. if tracking the north celestial pole, source $\Delta\theta$ away will move at an angular rate of $2\pi\Delta\theta/P$, $P \approx 23^{\rm h}56^{\rm m}04^{\rm s} \approx 86164$ s.

If correlator averaging time is long compared this apparent movement, the synthesized beam will broaden tangentially. To minimize this,

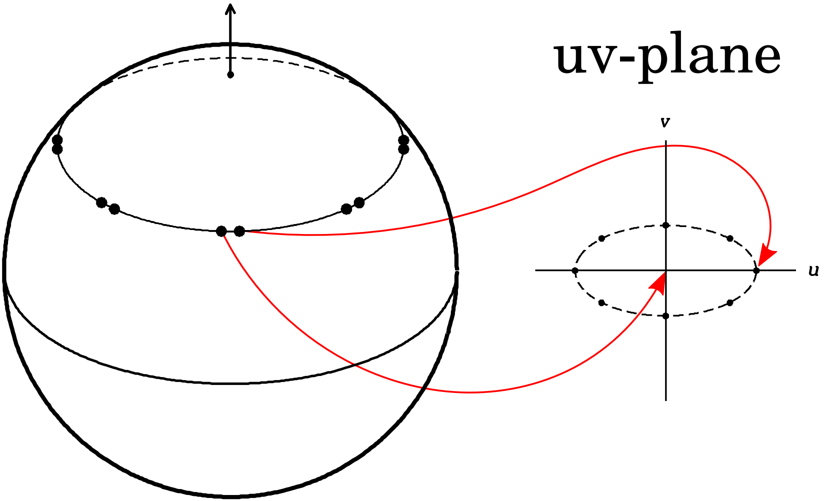
$$\Delta\theta\Delta t \ll \frac{\theta_{\rm s} P}{2\pi} \approx \theta_{\rm s} \times 1.37 \times 10^4 \text{ s}$$

Again: observe in slices, in this case short correlator integration time for one visibility data point.

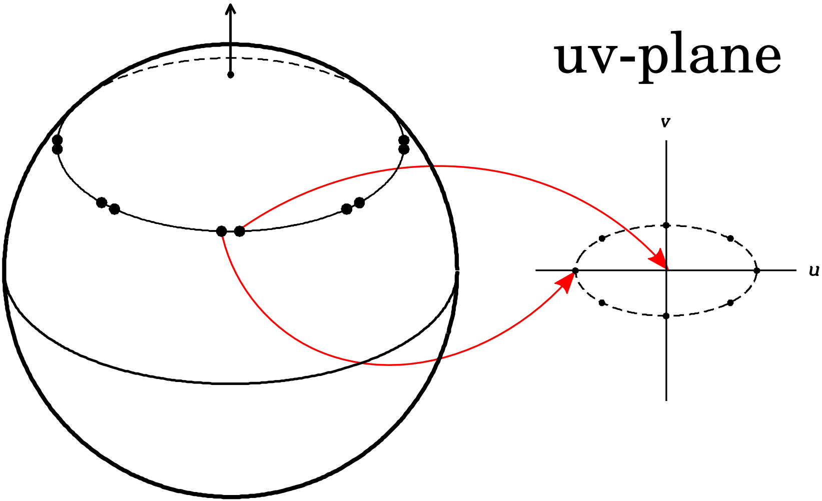



uv-plane
v
u

Earth rotation synthesis: source at $\delta + 30^\circ$



Earth rotation synthesis: source at $\delta + 30^\circ$



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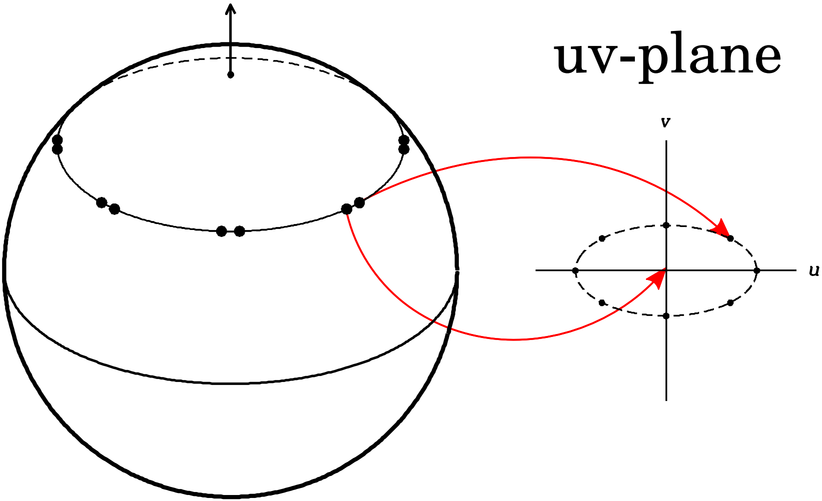

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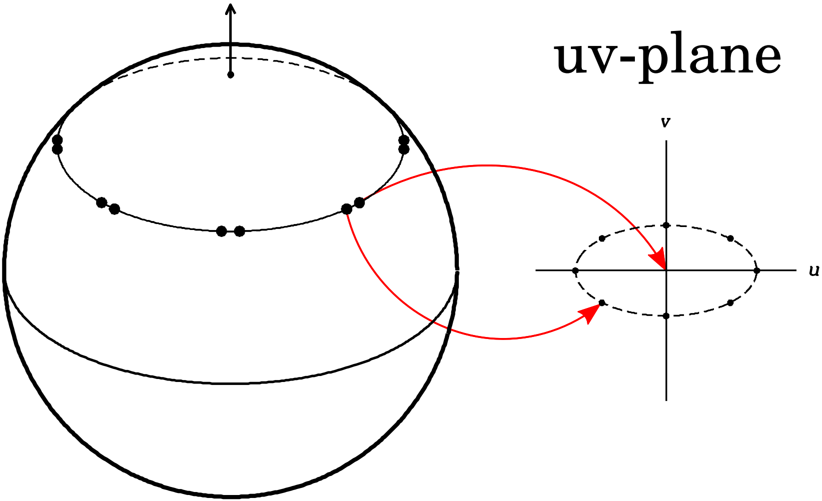

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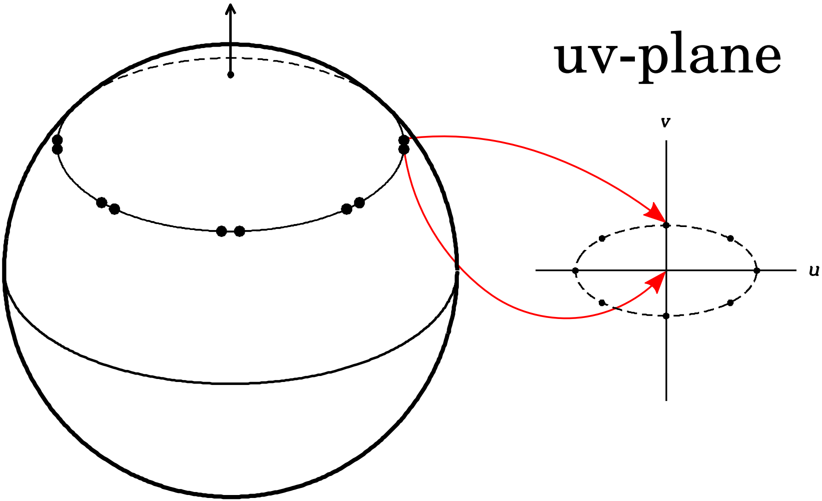

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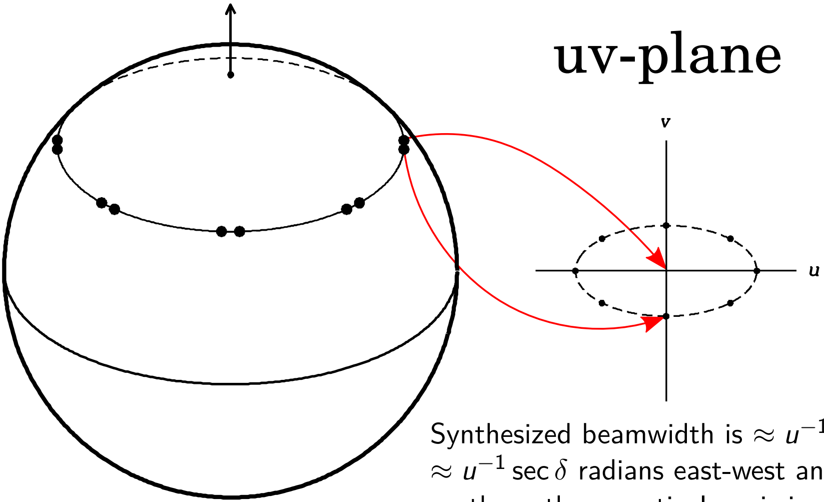
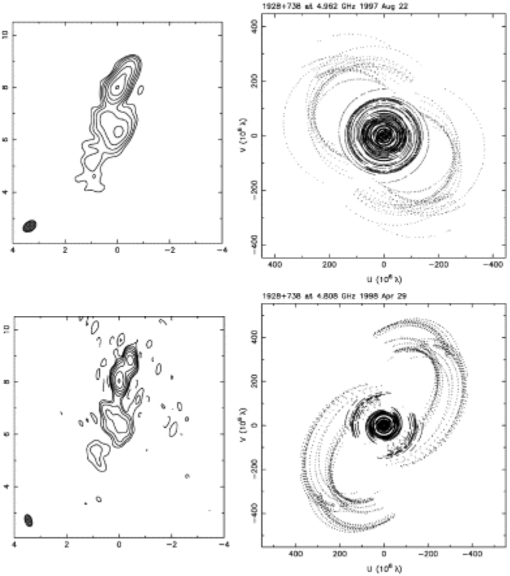


Synthesized beamwidth is $\approx u^{-1}$ and $\approx u^{-1} \sec\delta$ radians east-west and north-south respectively. u is in λ.

Example of uv-coverage



From Murphy et al. 1999

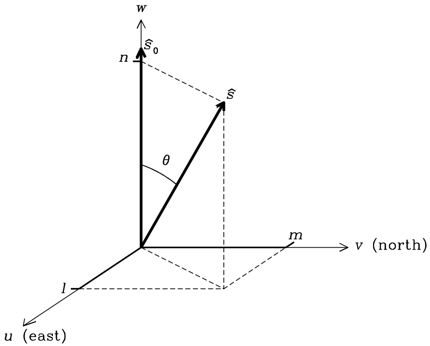
Coplanar interferometers

Very Large Array, New Mexico, USA. Y-shaped and nearly coplanar array of 27 25-m telescopes

Australia Telescope Compact Array, Narrabri, Australia. Six 22-m antennas (EW-NS).

In snapshot mode these interferometers can be treated as two-dimensional. But when using Earth rotation synthesis, even these interferometers fill a 3D volume.

Coordinate systems: uvw and lmn



The (u, v, w) coordinate system is used to describe any baseline vector $\vec{b}$ in three dimensions. The w-axis points usually to the source $\hat{s}_0$. The u and v axes point east and north and are in wavelenghts.

Unit vector directed to different parts of the source $\hat{s}$ has components (l, m, n), where $n = \cos(\theta) = (1 - l^2 - m^2)^{1/2}$. The components (l, m, n) are *direction cosines*.

Interferometers in Three Dimensions

In the (l, m, n) coordinates, solid angle equals

$$\mathrm{d}\Omega = \frac{\mathrm{d}l\,\mathrm{d}m}{(1 - l^2 - m^2)^{1/2}}. \tag{1}$$

Then the two-element interferometer visibility equation generalizes to

$$V_\nu(u, v, w) = \int\int \frac{I_\nu(l, m)}{(1 - l^2 - m^2)^{1/2}} \exp[-i2\pi(ul + vm + wn)]\,\mathrm{d}l\,\mathrm{d}m \tag{2}$$

Unfortunately this is not a 3D-FT.

Coplanar interferometer

If we assume that all antennas are on a plane, direction cosines are

$$l = \cos(\alpha), m = \cos(\beta), n = \cos(\gamma) = \sqrt{1 - l^2 - m^2} \quad (3)$$

$$\vec{b} = (\lambda u, \lambda v, \lambda w) = (\lambda u, \lambda v, 0), \quad (4)$$

where coordinates (u, v, w) are in wavelengths.
Then

$$\vec{b} \cdot \vec{s} \cdot \frac{\nu}{c} = ul + vm + wn = ul + vm \quad (5)$$

$$V_\nu(u, v) = \iint \frac{I_\nu(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-i2\pi(ul+vm)} \, \mathrm{d}l \, \mathrm{d}m \quad (6)$$

This is a 2-d FT between the projected brightness $I_\nu / \cos(\gamma)$ and the spatial coherence function i.e. visibility $V_\nu(u, v)$ that can be inverted:

$$I_\nu(l, m) = \cos(\gamma) \iint V_\nu(u, v) e^{i2\pi(ul+vm)} \, \mathrm{d}u \, \mathrm{d}v \quad (7)$$

Non-plane case

Non-plane case (measurement in volume instead on a plane):

$$V_\nu(u, v, w) = \iint \frac{I_\nu(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-i2\pi(ul + vm + wn)} \,\mathrm{d}l \,\mathrm{d}m \qquad (8)$$

Problem: not a 3d FT $\Rightarrow$ not easy to invert!

Solution: orient coordinates so that w-axis points to the source and use small angle approximation:

$$n = \cos\gamma = \sqrt{1 - \sin^2\gamma} = \sqrt{1 - \theta} \simeq 1 - \theta^2/2, \qquad (9)$$

where θ is the polar angle from image center to imaged source components.

Non-plane case, solution

Then

$$V_\nu(u,v,w) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-i2\pi(ul+vm+w\theta^2/2)}\,\mathrm{d}l\,\mathrm{d}m \quad (10)$$

If $w\theta^2 \ll 1$ or $\theta_{max} \lesssim \sqrt{\theta_{synth}}$, it can be neglected. E.g. for a 0.1 milliarcsecond synthesized beam the limit would be about 144 milliarcseconds. Then further

$$V'_\nu(u,v) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-i2\pi(ul+vm)}\,\mathrm{d}l\,\mathrm{d}m, \quad (11)$$

which is again a 2D FT, yippee!

Non-plane case, solution

Then

$$V_\nu(u,v,w) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-i2\pi(ul+vm+w\theta^2/2)}\, \mathrm{d}l\, \mathrm{d}m \qquad (10)$$

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which is again a 2D FT, yippee!

Larger fields can be mapped using *mosaicing*, i.e. observing several patches of sky separately. Because the new large interferometers (LOFAR, SKA) have very large primary beams that enable mapping of very wide field, these techniques are under fast development.

Nyquist-Shannon sampling theorem

To be able to use digital electronics and computers, the signal from antennas has to be *sampled*. This is done usually at IF (intermediate frequency)

If signal with bandwidth of $\Delta\nu$ is sampled at regular intervals of $\Delta t = \frac{1}{2\Delta\nu}$ the signal can accurately reconstructed. E.g. a signal with a bandwidth of 1 GHz must be sampled at least at $\frac{1}{2GHz} = 500$ ps intervals.

If sampling is sparser, out-of-band signals are *aliased* to the intended band creating interference and increasing noise.
Note: recently it has been proved that Shannon sampling theorem is a special case for *dense* signals, *sparse* signals can be recovered with fewer samples using **compressed sensing** methods.

Quantization

- The signal must be quantized during sampling. Digital audio uses at least 16-bit quantization i.e. $2^{16} = 65536$ levels.
- In interferomtery where the dynamic range is smaller and signals are noise-like much coarser quantization can be used ($1 - 2$ bits i.e. $2 - 4$ levels at the VLBA).
- Quantization increases noise: efficiency is 64% with two levels and 88% with four levels.
- Quantization creates systematic error in correlation which is a function of number of levels and degree of correlation. This error can be compensated using *Van Vleck* correction.

Correlation function

Correlation function:

$$C_{ij}(\tau) =< v_i(t)v_j(t+\tau) > \tag{12}$$

- If $v_i == v_j \Rightarrow$ autocorrelation.
- Both real and imaginary parts are needed in interferometry $\Rightarrow$ complex correlation.
- Complex correlation by correlating signals directly and with a 90° phase shift (Hilbert transform):

$$V_{ij}(\tau) =< v_i(t)v_j(t+\tau) > +i < \mathcal{H}[v_i(t)]v_j(t+\tau) > \tag{13}$$

Correlation in practice

Different correlator architectures:

- **Hardware correlators** using custom integrated circuits.
- **Software correlators** using COTS computers (COTS = 'Common Off The Shelf').
- FX and XF correlators (F = Fourier transform, X = multiplication).
- COTS CPU's getting faster ⇒ trend towards software correlators, also for flexibility.

Sensitivity I

Antenna response:

$$K = \frac{\eta_a A}{2k} 10^{-26} = \frac{A_{eff}}{2k} 10^{-26} = \frac{T_a}{S} \left[\frac{\mathrm{K}}{\mathrm{Jy}}\right] = \mathrm{DPFU} \tag{14}$$

System response **SEFD**: what amount of source flux increases the system noise as much as the noise of the receiving equipment when $T_a = 0$:

$$\mathrm{SEFD} = \frac{T_{sys}}{\mathrm{DPFU}} = \frac{2kT_{sys}}{A_{eff}} \cdot 10^{-26} \quad [\mathrm{Jy}] \tag{15}$$

Baseline sensitivity for antennas i and j (η_s = system efficiency):

$$\Delta S_{ij} = \frac{1}{\eta_s} \sqrt{\frac{\mathrm{SEFD}_i \cdot \mathrm{SEFD}_j}{2\Delta\nu\tau_{int}}} \quad [\mathrm{Jy}] \tag{16}$$

Sensitivity II

The number of baselines L is:

$$L = \frac{1}{2} \cdot N \cdot (N - 1), \tag{17}$$

where N = number of antennas.

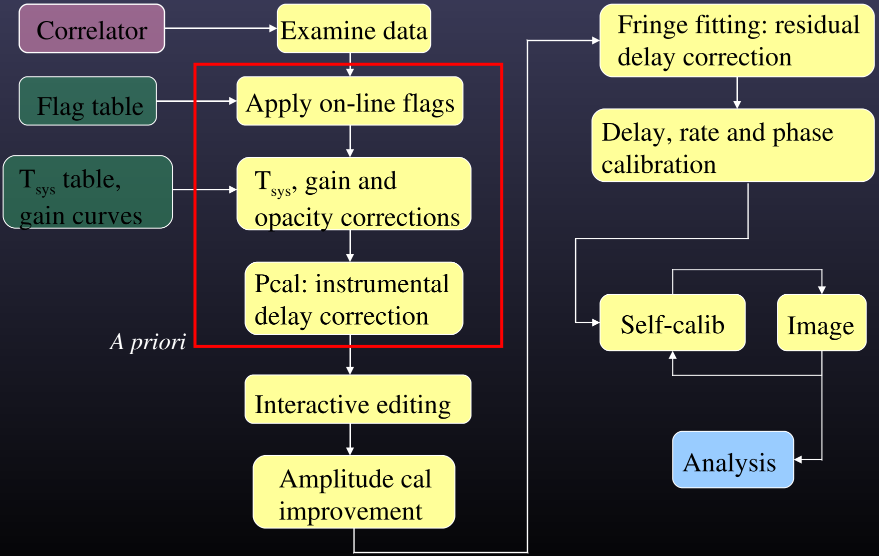
Image sensitivity I_m is standard deviation of mean of L samples (baselines),

$$\Delta I_m = \frac{1}{\eta_s} \sqrt{\frac{\mathrm{SEFD}_i \cdot \mathrm{SEFD}_j}{N(N-1)\Delta\nu\tau_{int}}} \quad [\mathrm{Jy/beam}] \tag{18}$$

Bias in visibility amplitude

- Note that while real and imaginary part of the visibility noise are Gaussian, amplitude and phase of the visibility are not.
- Especially amplitude is always positive $\Rightarrow$ for finite visibility noise mean is never zero $\Rightarrow$ bias!
- For zero SNR phase distribution is **uniform** $\Rightarrow$ can be used for weak signal detection.

VLBI data reduction path - continuum



From: Ylva Pihlström, Craig Walker, Tenth Synthesis Imaging Summer School, UNM 2006.

Apriori editing

- Flags from the on-line system will remove bad data from
  - Antenna not yet on source
  - Subreflector not in position
  - LO synthesizers not locked

Amplitude calibration: $\rho_{ij} \Rightarrow S_{ij}$

Raw correlator constant ρ_{ij} ($\propto V_{ij}$) must be calibrated to get **correlated flux densities**:

$$S^c_{ij} = \rho_{ij}\,\frac{b}{\eta_s}\sqrt{\frac{T^i_{sys}\,T^j_{sys}}{\mathrm{SEFD}_i\,\mathrm{SEFD}_j\,e^{-\tau_i}\,e^{-\tau_j}}}, \qquad (19)$$

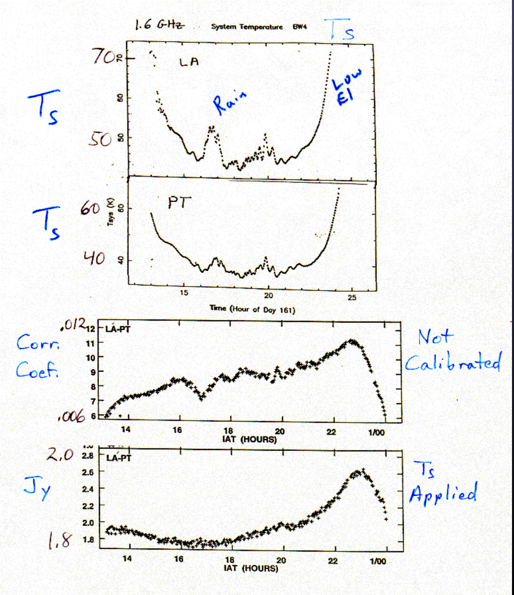
where

- ρ_{ij} = raw visibility
- b = correlator scaling factor
- η_s = system efficiency (digitization losses etc.)
- T^n_{sys} = system temperature at antenna n
- SEFD_n = system effective flux density at antenna n, incl. antenna gain vs. elevation
- $e^{-\tau_n}$ = atmospheric absorption at antenna n

Calibration with system temperatures

Upper plot: increased T_{sys} due to rain and low elevation

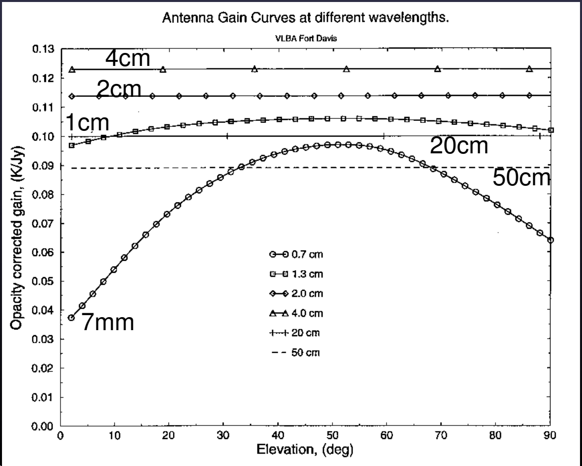
Lower plot: removal of the effect.



From: Ylva Pihlström, Craig Walker, Tenth Synthesis Imaging Summer School, UNM 2006.

VLBA gain curves

- Caused by gravitationally induced distortions of antenna
- Function of elevation, depends on frequency



From: Ylva Pihlström, Craig Walker, Tenth Synthesis Imaging Summer School, UNM 2006.

Atmospheric opacity I

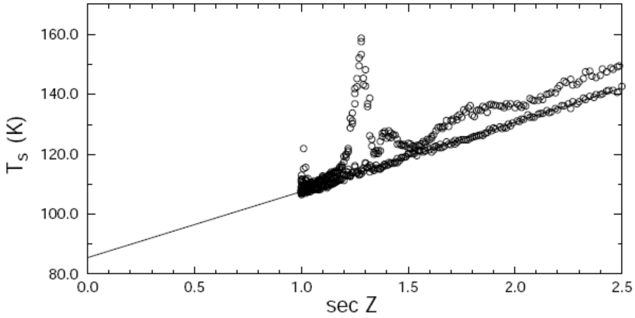
System noise temperature due to receiver and the atmosphere:

$$\begin{aligned} T_{sys} &= T_R + T_0(1 - e^{-\tau_0 \sec z}) \\ T_{sys} &\simeq T_R + T_0\tau_0 \sec z \end{aligned} , \qquad (20)$$

where T_R is receiver noise, T_0 is atmospheric effective temperature, τ_0 opacity towards zenith, and $\sec z$ is secant of zenith angular distance $\simeq$ line of sight airmass.

Atmospheric opacity II

T_{sys} plotted as a function of airmass ($T_{sys} \simeq T_R + T_0 \tau_0 \sec z$):



Zenith opacity τ_0 is the slope of the linear fit (times T_0) and $T_R = T_{sys}(\text{airmass} = 0)$

Figure from VLBA-book

Instantaneous opacity can be solved if T_R is assumed to be constant and known:

$$\tau = -\ln\left[\frac{T_0 + T_R - T_{sys}}{T_0}\right] \tag{21}$$

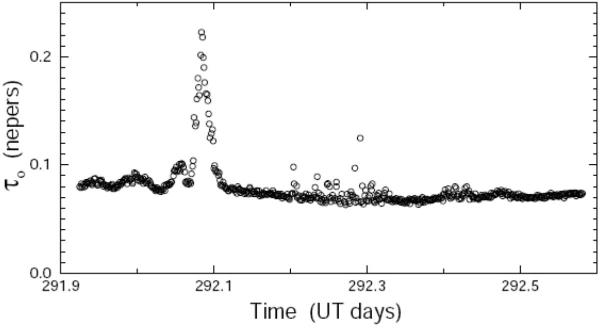
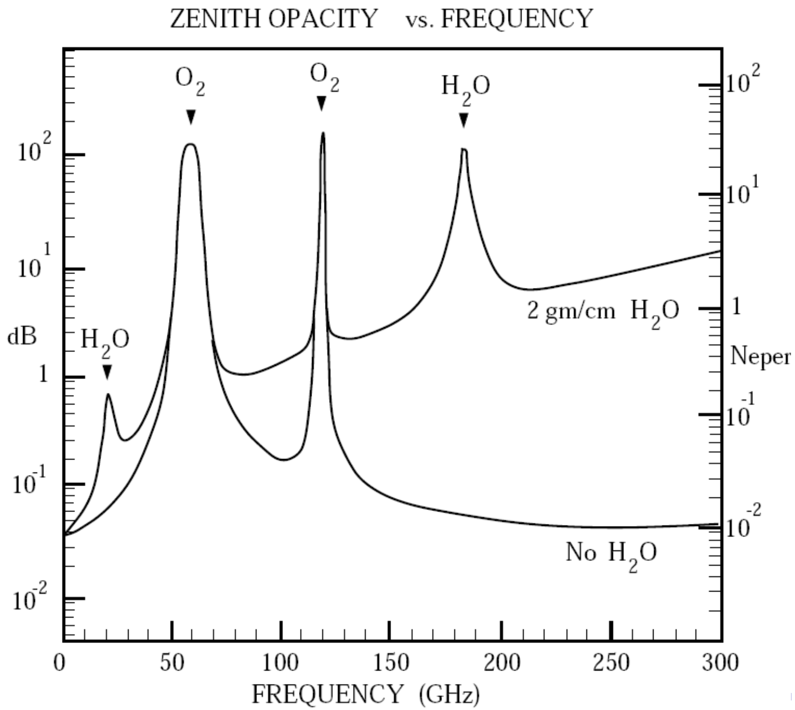


Figure from VLBA-book


ZENITH OPACITY vs. FREQUENCY
O_2
O_2
H_2O
H_2O
dB
Neper
2 gm/cm H_2O
No H_2O
10^2
10^1
1
10^{-1}
10^{-2}
0
50
100
150
200
250
300
FREQUENCY (GHz)