Relativistic Jets in Astrophysics – Problem set 1
Hand in your solutions by Monday 12.12.2011

1. A formula for apparent transverse speed on the plane sky of a relativistically moving emission region was derived in the lectures:

$$\beta_{\rm app} = \frac{\beta \sin\theta}{1 - \beta\cos\theta} \tag{1}$$

where $\beta = v/c$ is the true speed of the emission region and θ is the angle between the direction of motion and our line-of-sight. a) Show that for a given β the angle that maximizes $\beta_{\rm app}$ is $\sin\theta_{\rm c} = 1/\Gamma$ where $\Gamma = 1/\sqrt{1-\beta^2}$. b) Show that this maximum value is $\beta_{\rm app}^{\rm max} = \Gamma\beta$. c) You have measured both $\beta_{\rm app}$ and the relativistic Doppler factor $\delta = \Gamma^{-1}(1-\beta\cos\theta)^{-1}$ of a given jet. Now, calculate Γ and θ of that jet.

2. The Galactic microquasar GRS1915+105 shows a symmetric ejection of two radio emitting blobs with measured proper motions of $\mu_a = 17.6$ mas/day and $\mu_r = 9.0$ mas/day for the approaching and receding components, respectively. The source is at a distance of 12 kpc. Calculate the true speed of the blobs and the angle between the jet direction and our line-of-sight. Also calculate the expected ratio between the fluxes of approaching and receding components assuming an optically thin synchrotron spectrum with a power-law index $\alpha = -0.7$.

3. The brightness temperature of a source emitting intensity I_ν at frequency ν, is $T_b = \frac{c^2 I_\nu}{2k\nu^2}$, where c is the speed of light and k is the Boltzman constant. Using VLBI observations at 15 GHz, we measure the size of the radio emitting "core" at one end of the jet in blazar PKS 0420-014 by fitting a 2D Gaussian brightness distribution to the data and find that FWHM of 0.1 milliarcseconds gives the best fit. The same fit shows that core flux density $F_\nu = 5.8$ Jy. The source is at redshift of $z = 0.916$. Calculate the brightness temperature of the core in the rest frame of the source's host galaxy. (*Hint: Solid angle subtended by a 2D Gaussian* $\Omega = \frac{\pi\theta^2_{FWHM}}{4\ln 2}$. *And from cosmology* $I_\nu = I'_\nu/(1+z)^3$.) What can you say about the source based on its T_B? There is a well-known limit to the brightness temperature due to catastrophic cooling by inverse Compton radiation, $T_B^{lim} = 10^{12}$ K. What is the minimum relativistic Doppler factor needed to avoid IC-catastrophe in this source?

4. Blazars are highly variable sources and they show radio outbursts on time scales of months to years. Using the light-travel time argument, one can give an upper limit to the size of the emission region based on observed time scale of variability. From the observed variability time scale τ and peak flux density of the flare S_{max}, derive a formula for the lower limit of the "variability" brightness temperature $T_{B,var}$. Assume an optically thin spherical source and take into account cosmological distances (Remember from cosmology that angular distance d_a is luminosity distance d_L divided by $(1+z)^2$). What is $T_{B,var}$ if a source at $z = 1$ experiences a flare of 10 Jy in 20 days at 37 GHz? How does this “variability” brightness temperature transform between the rest frame of the emitting source and the observer’s frame in which the source moves relativistically?